

University of Waterloo
CS 860: Patterns in Strings

Fall 2008

Problem Set #3

Handed out Monday, November 10, 2008.

Due Monday, November 24, 2008 in class.

1. [5 marks] Show that there exists an infinite string over a finite subset of the integers with the property that of any four consecutive blocks of the same length, at least two of them have different sums.

2. [5 marks] Let α be a real number > 1 . Show that if there is an infinite word over a finite alphabet Σ avoiding α -powers, then there are infinitely many infinite words over Σ with that property.

3. [5 marks] Show that there exists an infinite string over a finite subset of the integers avoiding all subwords xx' with $|x| = |x'|$ and $\prod x = \prod x'$.

4. [10 marks] One possible definition of *abelian overlap* is a string of the form $xx'a$, where $|x| = |x'| > 0$, x' is a permutation of x , and a appears in x . (This is a generalization of an ordinary overlap, if we think of an overlap as “just slightly more than a square, and can be extended to a cube”.) Show that there is no infinite word avoiding this kind of abelian overlap over an alphabet of 3 letters, but that one exists over an alphabet of 4 letters.