



Kinetic Delaunay Triangulation

And Voronoi Diagram

Shahin Kamali



Plan..

- Introduction to Kinetic Data structures
- Kinetic Model for Delaunay Triangulation
- Applications
 - Lloyd's algorithm
 - RoboCup
- Project Demo



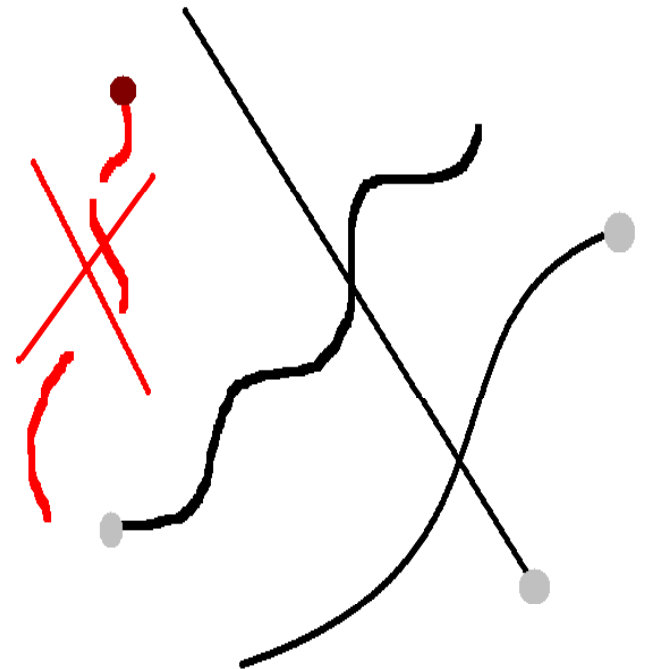
Motivation

- Maintain Delaunay Triangulation (Voronoi Diagram, etc.) for a set of objects undergoing continuous motion.
- Also useful for non-geometric problems...



Continuous Motions

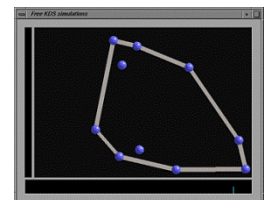
- Take advantage of the coherence present in continuous motion to process minimal number of combinatorial event.





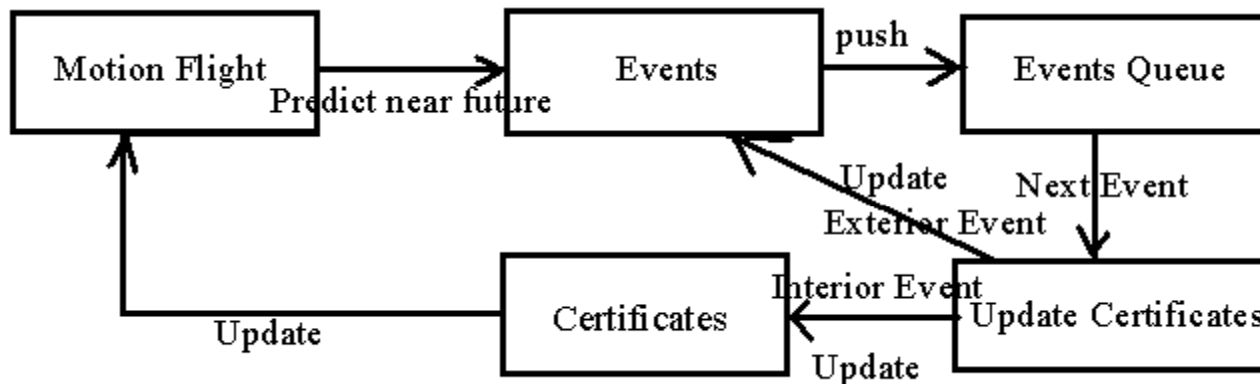
Kinetic Data Structures (KDS)

- Useful for process of discrete events associated with continuously changing data.
- Kinetisation:
 - Process of transforming an algorithm on static data into a data structure which is valid for continuously changing data.





Overview of Kinetic DS's





Motion Plan (Flight Plan)

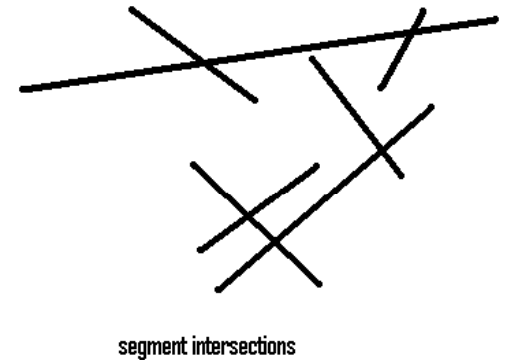
- We should know the behavior of agent in very short future.
- Examples:
 - Electrical Charges...
 - Molecules of Smoke !





Events in KDC

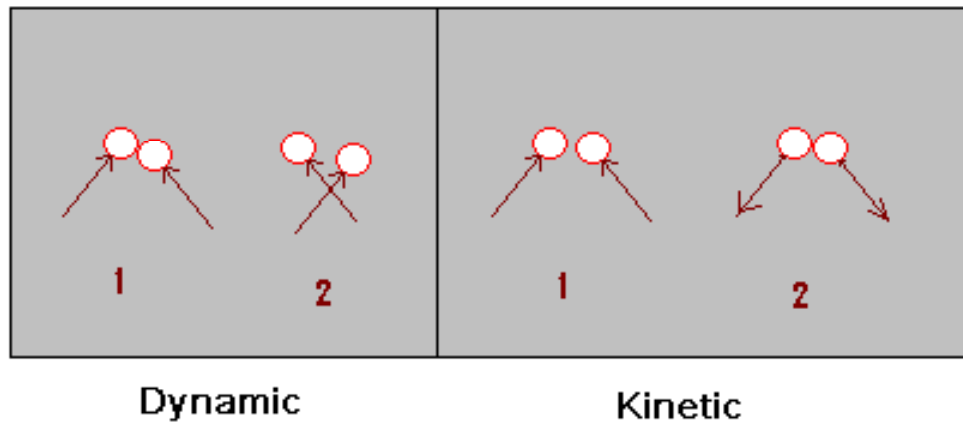
- Static Data structures
 - Support No “event”
 - Ex: arrays
- Dynamic Data Structures
 - Support Predictable events
 - Ex: trees (sweep line algorithms)
- Kinetic Data Structures
 - Support Unpredictable events (**online events**)





Online Events

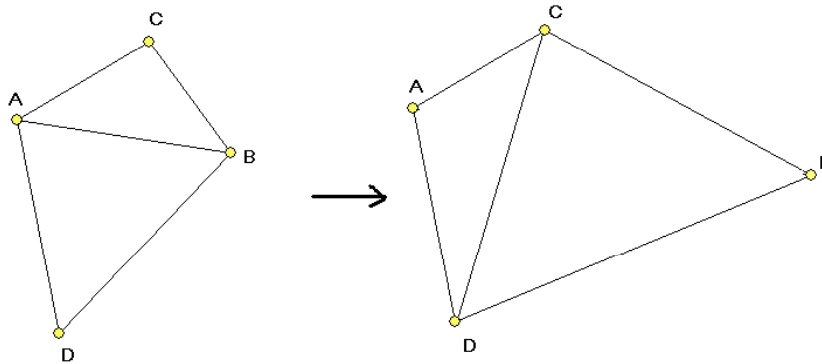
- Kinetic environments without online events can be handled by a Dynamic Data Structure.
 - Ex: ball movements





Certificates in KDS

- Elementary relations used in a Kinetic Data Structure:
 - In Delaunay Triangulation



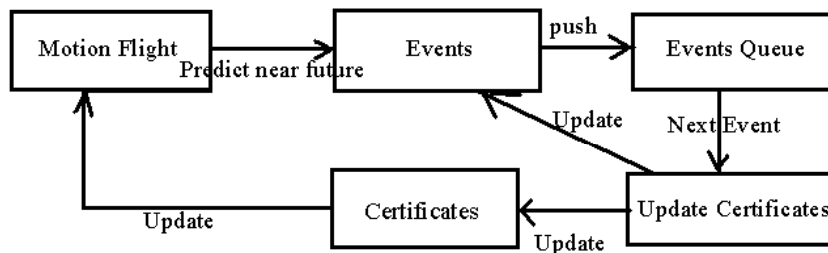
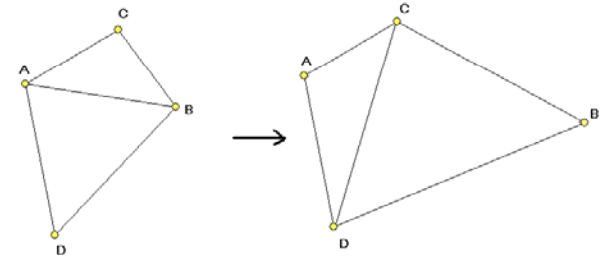
Before..
 $\text{isDT}(A,B,C)$
 $\text{isDT}(A,B,D)$

After..
 $\text{isDT}(B,C,D)$
 $\text{isDT}(A,C,D)$



Kinetic DS for Delaunay Triangulation

- Motion Flight : Depends on the problem
- Events:
 - Flip Edges
 - 3 collinear balls
 - Two balls crossing
 - A ball crosses the defined boundary





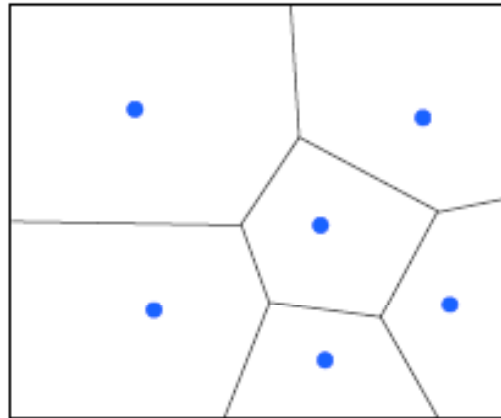
Kinetic DS for Voronoi Diagram

- Duality is valid for kinetic environments.
- We can use the algorithm proposed for Delaunay.
- Best Time algorithm for Delaunay (and Voronoi) $n^{7/3}$ [which is not that good]
- Proposing a better algorithm is challenging.



Centroidal Voronoi Diagram

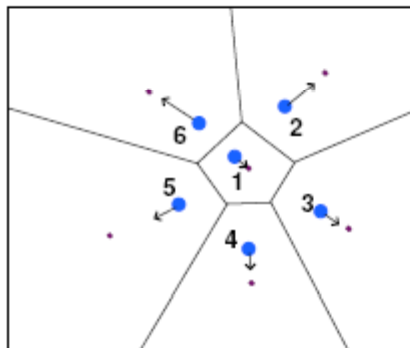
- A Voronoi Diagram for which each site is in the centroid of its cell
- Centroid \rightarrow center of mass



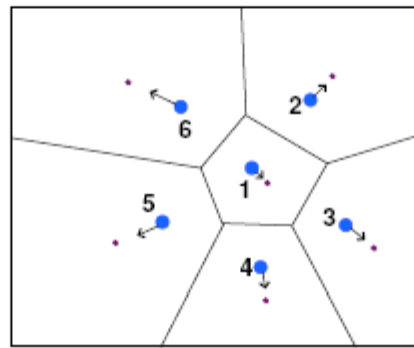


Lloyd's algorithm

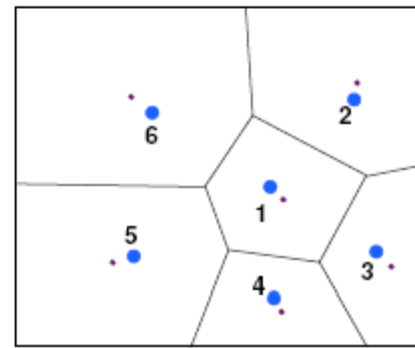
- Just Compute Voronoi diagram and in each sense (time slide) force sites toward their centroids.
- Do we get an equilibrium in this way?
 - - a challenging question
- No effective way to implement this algorithm (for large data)



a



b

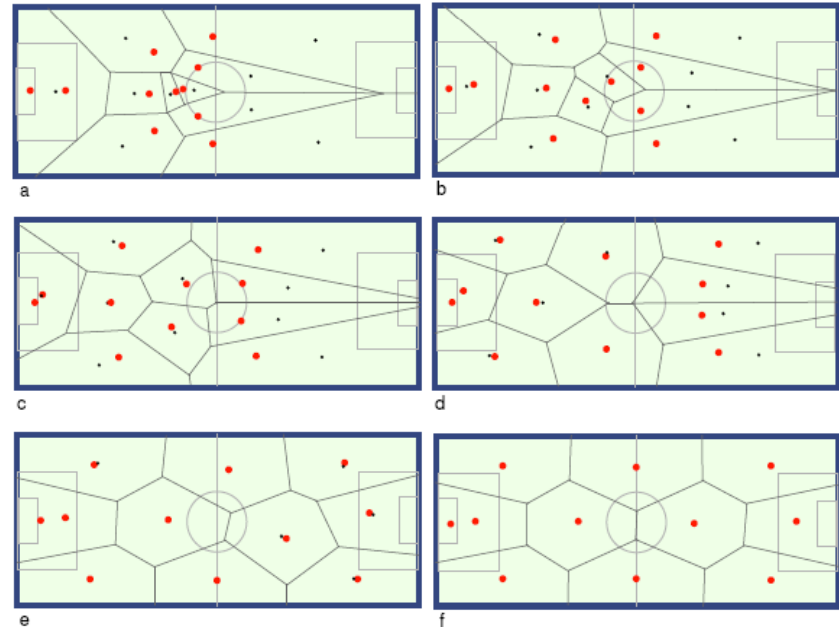


c



RoboCup Simulation

- Apply Lloyd Algorithm for positionig of agents.





Thanks...