Your Name: .

Student Number: .

Final Examination Sample Questions

Course: CS360: Introduction to the Theory of Computing

Term:

Date:

Duration: 3 hours.

NO ADDITIONAL MATERIALS ALLOWED.

Understanding the questions is part of the examination. The proctors will not answer any questions. However, if you think there is a problem with one of the questions, bring it to the proctor's attention.

Indicate clearly how you have obtained your answers, show all the necessary steps, but be concise.

If necessary, use the back of the *previous* page to continue your answer.

1. (16 marks) **Finite Automata and Regular Languages**.

   (a) (6 marks) Give a formal definition of a *deterministic finite automaton.*

   (b) (1 mark) Define a *configuration of a deterministic finite automaton.*

   (c) (2 mark) Define the *yields in one step relation* $\vdash_M$ for a deterministic finite automaton M.

   (d) (1 mark) Define the concept of a string w *accepted* by a deterministic finite automaton.

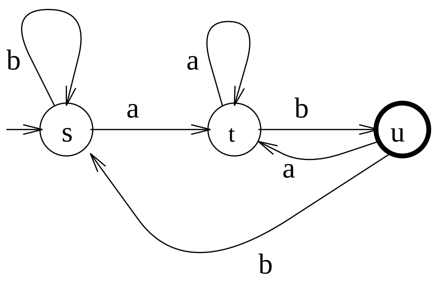

Figure 1: Finite Automaton

(e) (6 marks) For the automaton M of Figure 1, prove, by induction on the length of w, that, for every $q \in K$ and $w \in \Sigma^* ab$,

$$(q, w) \vdash_M^* (u, \varepsilon).$$

Separate your proof clearly into: 1) Basis, 2) Inductive Assumption, and 3) Inductive Step.

2. (18 marks) Context-Free Grammars.

(a) (3 marks) Let $G = (V, \Sigma, R, S)$ be defined as follows: $V = \{S, a, b\}, \Sigma = \{a, b\}$, and the set of productions is

$$S \to aSSb \mid Sa \mid \varepsilon$$

Find all the strings of length three generated by G.

(b) (2 marks) Find a parse tree for G that generates the string $aabab$.

(c) (2 marks) Find a grammar G' for the language $L(G) - \{\varepsilon\}$.

(d) (2 marks) Find a grammar G'' in Chomsky normal form such that $L(G'') = L(G')$.

(e) (3 marks) State the pumping theorem for context-free languages. Use the (stronger) version that was done in class.

(f) (6 marks) Using the pumping theorem of part (d), prove that the language $L = \{a^i b^{i+1} c^{i+2} : i \geq 0\}$ is not context-free.

3. (18 marks) Turing machines. Marking of these questions will also depend on the *simplicity* of your designs. Except for (a), you can use some of the following Turing machines or the machines you have designed as subroutines and combine them together: σ (symbol writer), for each $\sigma \in \Sigma$, R (move right), L (move left), $L_{\#}$ (move left till $\#$), $R_{\#}$ (move right till $\#$), C (copy), S_L (transform $x\#w\underline{\#}$ to $xw\underline{\#}$, independent of $x \in (\Sigma \cup \{\#\})^*$), S_R (symmetric to S_L, shift to right).

(a) (6 marks) Design a nondeterministic Turing machine with only 2 states (in addition to h) accepting $L = \{w : \ w \in \{a, b\}^* \text{ and } w \text{ contains } aa \text{ as a substring}\}$. Draw the state diagram. (Do not use other Turing machines as subroutines.)

(b) (4 marks) Recall that the characteristic function of a language L is defined as follows: $\chi(w) = Y$ if $w \in L$ and $\chi(w) = N$ of $w \notin L$. Construct a Turing machine that computes the characteristic function for ϕ, the empty set.

(c) (4 marks) Construct a Turing machine that computes $f(m, n) = m + n$, where integers are represented in unary notation (input m, n is given as $\#I^m\#I^n\underline{\#}$).

(d) (4 marks) Construct a Turing machine that computes $f(n) = 3n$, where integers are represented in unary notation (input is $\#I^n\underline{\#}$).

4. (18 marks) Undecidability.

   (a) (3 marks) Give a formal statement of the halting problem for Turing machines.

   (b) (2 marks) Show that if L is Turing-decidable, then $\overline{L}$ is also Turing-decidable.

   (c) (5 marks) Show that it is decidable to determine, given a Turing machine M, whether M moves at least 17 steps when started on the empty tape.

   (d) (8 marks) Show that it is undecidable to determine, given a Turing machine M, whether M ever writes "go raptors" (assuming that M's alphabet includes letters g, o, r, a, p, t, r, s) when started on the empty tape (as it is undecidable to determine if Raptors will ever win an NBA title).

5. (15 marks) Answer true or false for the following questions. No explanation is needed.

(a) If a language contains only 5 elements, then it is regular.

(b) $\{x \in \{a, b\}^* : \ |x| \leq 1,000,000\}$ contains too many elements; hence it is not regular.

(c) Every NFA can be converted to an equivalent DFA with same number of states.

(d) If L is regular, then it is Turing decidable.

(e) The class of context-free languages is not closed under intersection.

(f) A language L is Turing decidable iff L Turing acceptable.

(g) $\{a, aa\} \cap \phi = \{a, aa\}$, where ϕ is the empty set.

(h) Let Σ be nonempty. If $L \subset \Sigma^*$ is a finite set, then $\overline{L} = \Sigma^* - L$ is infinite.

(i) Let Σ be nonempty. If $L \subset \{0, 1\}^*$ is infinite, then $\overline{L} = \Sigma^* - L$ is finite.

(j) $\{ww^Rw | w \in \{0, 1\}^*\}$ is not a CFL, but can be decided by a Turing machine.

(k) If L can be accepted by a 2-tape deterministic Turing machine in $T(n)$ steps, then it can be accepted by a 1-tape deterministic Turing machine in $O(T^2(n))$ steps.

(l) If L is regular, then L can be accepted by a 1-tape deterministic Turing machine in linear, *i.e.* $O(n)$, time.

(m) It is possible to write a computer program (say in C++) which takes any C++ program as input and outputs "yes" if the program will stop on input 0, and outputs "no" otherwise.