Assignment 5. CS341, Winter 2011

Distributed Tuesday, March 15, due March 29, 1pm, 2011. Hand in to the assignment boxes on the 3rd floor of MC, before 1pm.

- 1. (15 marks) Prove, by reduction, that the following problems are undecidable.
 - Given a program P, decide if P halts on some inputs. That is, there is an input w such that P halts on w.
 - Given a program P, decide if P outputs "You got an A for CS341" on input "What is my CS341 score?".
 - Given program P, decide if the set of inputs accepted by P is finite.

Hint: reduction from the Halting problem.

- 2. (20 marks) Prove that the follow problems are NP-complete.
 - The longest path problem: Given a weighted undirected graph G, integer k, and vertices u, v both in G, is there a simple path (i.e. no vertex repeated) in G from u to v of length at least k? (Hint: reduction from the Hamiltonian cycle problem.)
 - The independent set problem. In a simple graph G, an independent set is a subset of its vertices that are pairwise not adjacent. In other words, the subgraph induced by these vertices has no edges, only isolated vertices. Then, the independent set problem asks: given a graph G and a positive integer k, does G have an independent set of cardinality at least k?
- 3. (15 marks) Given an undirected graph G = (V, E) and a set of vertices $L \subseteq V$, determine which of the following problems are NP-complete and which are in P. Give proofs.
 - (a) Is there a spanning tree of G whose leaves include the set L?
 - (b) Is there a spanning tree of G whose leaves are precisely the set L?
 - (c) Is there a spanning tree of G whose leaves are a subset of L?