CS341 Assignment 4 Marking Scheme

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1 Question 1

a)

For clarity, we will ignore additive constants in this analysis. If we partition elements into groups of 3, we have $\frac{n}{3}$ such groups. Let $M = m_1, ..., m_{n/3}$ be the medians of these groups and let m be the median of the set M. There are $\frac{n}{6}$ groups whose median is below m. In each such group, there are at least 2 elements that are at most m. Therefore, the number of elements in the array that are at most m is at least $\frac{2n}{6} = \frac{n}{3}$. Hence the number of elements that are greater than m is at most $\frac{2n}{3}$. Analogously, we can show that the number of elements that are less than m is at most $\frac{2n}{3}$. Hence the recursion becomes

$$T(n) = T(r) + T(n/3) + dn$$

where $r \leq \frac{2}{3}n$.

This is not linear.

b)

Analogously to a), we can show that the recursion becomes

$$T(n) = T(r) + T(n/7) + dn$$

where $r < \frac{5}{7}n$. This is linear. To prove this, we find values c, n_0 such that $T(n) < cn$, for all $n > n_0$.

$$c\frac{5}{7}n + c\frac{n}{7} + dn < cn$$

$$c\frac{6}{7}n + dn < cn$$

This is satisfied when

$$c > 7d$$

for all $n > 0$.

Marking scheme: 5 marks for each question. In question a): 2 marks for the recursion, 1 mark for the derivation, 2 marks for saying it is not linear. In question b): 1 mark for the derivation, 2 marks for the recursion, 2 marks for proof of linearity.

2 Question 2

When the algorithm checks a value in the array, the adversary always reveals a value other than 1. If the algorithm stops after asking fewer than n comparisons and claims the array does not contain a 1, the adversary sets one of the unchecked elements of the array to 1. If the algorithm says an array contains a 1, the adversary sets all the remaining values to 0. Hence, it takes n comparisons to check if an array of length n contains a 1.

Marking scheme: 5 marks for the question. Marks deducted for arguments that were not adversarial.

3 Question 3

There are many ways to solve this problem in linear time. One key observation is that in this model it takes $O(n)$ time to compute the degree of a vertex, since it requires looking up the whole row of a matrix. Therefore, it is important to only compute the degree of a constant number of vertices.

checkBody(v):
if(degree(v)!=n-2) return false
find vertex u that is not a neighbour of v
if(degree(u)!=1) return false
find the unique neighbour w of u
return degree(w)==2

checkScorpion(V,E):
pick vertex $v \in V$
if(degree(v)==n-2) return checkBody(v)
if(degree(v)==2){
let u_1, u_2 be the neighbours of v
return checkBody(u_1) or checkBody(u_2)
}
if(degree(v)==1){
let u be the neighbour of v
if(degree(u)==2)){
let u_1 be the neighbour of u that is not v
return checkBody(u_1)
}

if(degree(u)==n-2){
return checkBody(u)
}
return false
}
let N be the set of neighbours of v
let $S \leftarrow V \backslash N$
pick $s \in S, n \in N$
while(N, S are not empty) {
if $(n, s) \in E$ {
$S \leftarrow S - \{s\}$
pick $s \in S$
}
if $(n, s) \notin E$ {
$N \leftarrow N - \{n\}$
pick $n \in N$
}
}

if(degree(s)==1){
let u be the neighbour of s
if(degree(u)==2)){
let u_1 be the neighbour of u that is not v
return checkBody(u_1)
}
}
return false

Marking scheme: 2 marks for each of the cases where deg(v)=1,2,n-2. 4 marks for checking if a vertex is a foot. In general, $O(n^2)$ solutions could get at most 2 marks, except in cases where only one of the special cases was $O(n^2)$, in which case the maximum was 5 marks. Marks were deducted for lack of clarity in extreme cases.

4 Question 4

Suppose that $\bar{A}$ is solvable. That means there exists an algorithm $Solve(w)$ that, given a word w, outputs "yes" if $w \in \bar{A}$ and "no" if $w \notin \bar{A}$. Consider the following algorithm:

SolveA(w):
if(Solve(w)=="yes") return "no" return "yes"

This algorithm determines outputs "yes" if and only if $w \in A$. This contradicts the unsolvability of A, which ends the proof.

4 Question 4

Marking scheme: 10 marks. Up to 4 marks deducted for clarity. A completely incorrect argument using a proof by contradiction could get at most 2 marks.