CS360 Fall 1999

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Assignment 4, Due Date: Dec. 5

1. Design a (deterministic) 1-tape Turing machine, using the "machine schema", to do pattern-matching. That is, for input $\#x\#y\underline{\#}$, $x, y \in \{a, b\}^+$; the output is $\#Y\underline{\#}$ if $y = uxv$ for some u, v (i.e., x is a substring of y), and otherwise the output is $\#N\underline{\#}$. (Note, you do not have to design an efficient TM, such as the linear time algorithms; just try to design a simple one.)

2. Design nondeterministic Turing machines (do not use machine schema) accepting the following languages using state diagrams. You must try to use as few states as possible.

   (a) a^*bb^*a.

   (b) $\{a^ib^jc^k \;:\; i = j \text{ or } i = k\}$.

   (c) $\{xcy \;:\; x, y \in \{a, b\}^*,\; y = uxv, \text{ for some } u, v\}$. This is the same pattern-matching language as in Problem 1.

3. Do Problem 9.38 (page 291) in the textbook. (Please read Section 9.7 for this problem.) This question will not be marked.

4. Prove the following two claims. Notice the difference between the two proofs. (a) is simpler, and (b) requires dovetailing style simulation.

   (a) Let L_1 and L_2 be Turing acceptable (or enumerable). Prove that $L_1 \cap L_2$ is also Turing acceptable (or enumerable).

   (b) Let L_1 and L_2 be Turing acceptable (or enumerable). Prove that $L_1 \cup L_2$ is also Turing acceptable (or enumerable).

5. Do Problem 10.4 (page 307) in the textbook.

6. Show that the language

$$L_d = \{\rho(M) \mid L(M) \text{ is Turing-decidable}\}$$

   is not Turing-acceptable. (Hint: Since the halting problem is Turing-acceptable, but not Turing-decidable, the complement of the halting problem is *not* Turing acceptable. You can carefully use that problem to do reduction. Note that the following approach fails: if we could show that $\overline{L_d}$ is Turing-acceptable but not Turing-decidable, then it would follow that L_d is not Turing-acceptable. This approach has to fail because it is known that $\overline{L_d}$ is also not Turing-acceptable.)