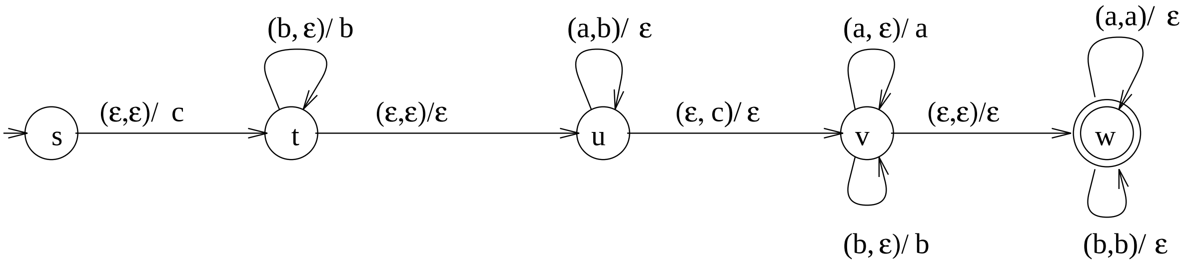

Figure 1: A pushdown automaton M

CS360 Fall 1999

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Assignment 3, Due Date: Nov. 8, 1999

1. Let $L = \{w\tilde{w}^R : w \in \{a,b\}^*$ and $\tilde{w}$ is w with a's replaces by b's and b's by a's $\}$. Find a PDA accepting L.

2. The pushdown automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined as follows: $K = \{s, t, u, v, w\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, c\}$, $F = \{w\}$, and Δ is given by the state diagram in Figure 1.

   (a) Find the four shortest strings in $L(M)$.

   (b) Describe $L(M)$ in words.

   (c) Find a context-free grammar for $L(M)$.

3. Consider the grammar $G = (V, \Sigma, R, S)$ with $V - \Sigma = \{S, A, B, C\}$, $\Sigma = \{a, b, c\}$ and R given by: $S \to aAb$, $\quad A \to B|aAb|AA$, $\quad B \to c|C$, $\quad C \to \varepsilon$.

   (a) Find a grammar G' such that $L(G') = L(G)$, and G' has no rules of the form $X \to \varepsilon$.

   (b) Find a grammar G'' such that $L(G'') = L(G')$ and G'' has no rules of the form $X \to Y$, $X, Y, \in V - \Sigma$.

   (c) Find a grammar G''' such that $L(G''') = L(G'')$ and G''' is in Chomsky normal form.

4. Prove that there is an algorithm which will decide whether for a given context-free grammar G, $L(G)$ is finite.

5. Let $L_1 = \{a^n b^n : n \geq 0\}$, and $L_2 = b^* a^*$, $L_3 = \{ww^R : w \in \{a, b\}^*\}$, and let $L = (L_1 L_2) \cap L_3$. Is L a context-free language? Prove your answer.