Regular Languages

A language **L** is regular if one of the following is true:

Regular Expression	Language
Ø	**L** = Ø
ε	**L** = {ε}
a	**L** = {a} for some a ϵ ∑
r_1*	**L** = $\mathbf{L_1}$* for some regular language $\mathbf{L_1}$
r_1 r_2	**L** = $\mathbf{L_1}$ $\mathbf{L_2}$ for some regular languages $\mathbf{L_1}$, $\mathbf{L_2}$
r_1 + r_2	**L** = $\mathbf{L_1} \cup \mathbf{L_2}$ for some regular languages $\mathbf{L_1}$, $\mathbf{L_2}$

If ***r*** is a regular expression, then **L**(***r***) is the language of ***r***.

Basic Identities:

Øe = eØ = Ø	Any e concatenated with Ø is Ø
Ø* = ε	Convention: Any language L^0 = {ε}
ε* = ε	
x + x = x	Union
(x*)* = x*	Kleene * is closed
x(y + z) = xy + xz	Distribution

Regular languages are *closed* under Kleene *, union and concatenation.

A class of languages is *closed under a binary (unary) operation* if applying that operation to 2 (1) languages in the class always yields a language in the class.

Deterministic Finite Automata

- Simplest computers
- Finite memory, finite program
- Given a finite input word, it moves from one state to another
    - Each move is based on an input symbol
    - At the end of the input, it either accepts or rejects

Limitations

- Cannot look back in their input

Definition – 5 Parameters: **($Q, \Sigma, q_0, \delta, A$)**

- **Q** = set of computation states
- **∑** = finite input alphabet
- q_0 = start state
- **δ** = transition function
- **A** = accept states

Let FA (finite automata) **M** = **($Q, \Sigma, q_0, \delta, A$)** and w ϵ ∑*. We say "FA **M** accepts *w*" if starting at q_0 and following the transition function **δ** for each letter in *w*, in turn, ends up in a state from **A**.

Language of the finite automation **M**: all words accepted by **M**.

Formally, acceptance of a word: **M** = **($Q, \Sigma, q_0, \delta, A$)** accepts *w* exactly when $\delta^*(q_0, w) \in A$.

Language of the FA: all such words

- **L**(**M**) = $\{w \in \Sigma^* \text{ where } \delta^*(q_0, w) \in A\}$
- or **L**(**M**) = {*w* ϵ ∑* where **M** accepts *w*}

L(**M**) = language of the FA **M** or language *accepted*/*recognized* by FA **M**.