Induction:

- state you are using induction and on what.
- Basis - show it works for first k cases
  - often $k=1$
- Induction Hypothesis
- Statement to prove - what we want to prove
- Induction Step - the proof

Weak Induction

In the hypothesis we assume true for ℓ and show true for $\ell+1$.
Then it is true for all $\ell \geq k$.

Strong Induction

In the hypothesis we assume true for all m between k and ℓ and show true for $\ell+1$.
Then it is true for all $\ell \geq k$.

Structural Induction

If hypothesis is true for simple objects, and its true for how we combine them, then its true for all objects, too.

Example:

$$rev(xy) = rev(y)\,rev(x)$$

Recall: $rev(x)$ is x written backwards

Proof by Induction on $|y|$:

Base case: $|y| = 0$ then $y = \varepsilon$

$$rev(xy) = rev(x \cdot \varepsilon) = rev(x)$$
$$= rev(\varepsilon)\,rev(x) = rev(y)\,rev(x) \quad \square$$

Induction Hypothesis: Assume $rev(xy) = rev(y)\,rev(x)$ for y of length $k-1$.

Need to show: $rev(xy) = rev(y)\,rev(x)$ for y of length k.

Induction Step:

Let $y = za$ for a single character $a \in \Sigma$

Then $xy = xza$ and

$$rev(xy) = \underbrace{rev((xz)a) = a\,rev(xz)}_{\text{by the Induction Hypothesis since } |a| < k}$$

$= a\,rev(z)\,rev(x)$ ⇒ by I. H. since $|z| < k$

$= rev(a)\,rev(z)\,rev(x)$

$= rev(za)\,rev(x)$

$= rev(y)\,rev(x) \quad \square$

Recursive Structure

- many objects are defined recursively
  - Fibonacci Numbers
  - Sequences of letters
  - Binary Trees
  - Many things built using Scheme Data Definitions

Structural Induction Example

Let L be the smallest language over $\Sigma = \{+, a, (,)\}$ that satisfies

1. $a \in L$
2. If w_1 and w_2 are in L, so is $(w_1 + w_2)$

Theorem: No string in L contains)(.

Proof by structural induction.

Base case: a does not contain)(.

Induction case:

Suppose w_1 and w_2 do not contain)(.
Must show $(w_1 + w_2)$ does not contain)(.

Proof of Induction:

Show that no) in $(w_1 + w_2)$ comes right before a (.

Cases ① the last) has nothing after it.

② no) in w_2 comes directly before a (.

- or that would break the I. H.
- Similarly, for w_1

③ the last) in w_1 may come before + but not a (.

④ the last) in w_2 may come before) but not a (.

Since that is all of the)'s in $(w_1 + w_2)$,

no member of L has)(in it. □