Introduction to Mechanism Design

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October 2, 2006

Outline

Example: Building a Pool

- Cost of building the pool is \$300
- If together all agents value the pool more than \$300 then it will be built
- Clarke Mechanism
  - Each agent announces v_i and if $\sum_i v_i \geq 300$ then it is built
  - Payments $t_i = \sum_{j \neq i} v_j(x^{-i}, v_j) - \sum_{j \neq i} v_j(x^*, v_j)$

Assume $v_1 = 50$, $v_2 = 50$, $v_3 = 250$. Clearly, the pool should be built.
Transfers: $t_1 = (250 + 50) - (250 + 50) = 0 = t_2$ and
$t_3 = (0) - (100) = -100$.

Pros

- Social welfare maximizing outcome
- Truth-telling is a dominant strategy
- Feasible in that it does not need a benefactor ($\sum_i t_i \leq 0$)

Cons

- Budget balance not maintained (in pool example, generally $\sum_i t_i < 0$)
  - Have to burn the excess money that is collected

Theorem

Let the agents have quasilinear preferences $v_i(x, \theta_i) - t_i$ where $v_i(x, \theta_i)$ are arbitrary functions. No social choice function that is (ex post) welfare maximizing (taking into account money burning as a loss) is implementable in dominant strategies. [Laffont&Green 79]

- Vulnerable to collusion (even with coalitions of just 2 agents).

Bayes-Nash Implementation

- Goal is to design mechanisms so that in **Bayes-Nash** equilibrium s^*, the outcome is $f(\theta)$.
- Weaker requirement than dominant-strategy implementation
  - An agent's best response strategy may depend on others' strategies
    - Agents may benefit from counterspeculating
- Can accomplish more under with Bayes-Nash implementation than dominant strategy implementation
  - Budget balance and efficiency under quasi-linear preferences

Expected Externality Mechanism

d'Aspremont&Gerard-Varet 79, Arrow 79

- Similar to Groves mechanism but the transfers are computed based on agent's revelation v_i, *averaging over possible true types of the others* v^*_{-i}
- Outcome: $x(v_1, \ldots, v_n) = \arg\max_x \sum_i v_i(x)$
- **Others' expected welfare when agent i announces v_i**

$$\xi(v_i) = \int_{v_{-i}} p(v_{-i}) \sum_{j \neq i} v_j(x(v_i, v_{-i}))$$

This measures the change in expected externality as agent i changes its revelation

d'AGVA Mechanism

Theorem

Assume that agents have quasi-linear preferences and statistically independent valuation functions v_i. Then the efficient SCF f can be implemented in Bayes-Nash equilibrium if

$$t_i(v_i) = \xi(v_i) + h_i(v_{-i})$$

for arbitrary function $h_i(v_{-i})$.

Unlike in dominant-strategy implementation budget balance is achievable

- Set $h_i(v_{-i}) = -\frac{1}{n-1}\sum_{j\neq i}\xi(v_j)$

d'AGVA does not satisfy participation contraints

- An agent might get higher expected utility by not participating

Participation Constraints

We can not force agents to participate in the mechanism. Let $\hat{u}_i(\theta_i)$ denote the (expected) utility to agent i with type θ_i of its outside option.

- **ex ante individual-rationality**: agents choose to participate before they know their own type

$$E_{\theta\in\Theta}[u_i(f(\theta),\theta_i)] \geq E_{\theta_i\in\Theta_i}\hat{u}_i(\theta_i)$$

- **interim individual-rationality**: agents can withdraw once they know their own type

$$E_{\theta_{-i}\in\Theta_{-i}}[u_i(f(\theta_i,\theta_{-i}),\theta_i)] \geq \hat{u}_i(\theta_i)$$

- **ex-post individual-rationality**: agents can withdraw from the mechanism at the end

$$u_i(f(\theta),\theta_i) \geq \hat{u}_i(\theta_i)$$

Summary

Impossibility and Possibility Results

- **Gibbard-Satterthwaite**
  - Impossible to get non-dictatorial mechanisms if using dominant-strategy implementation and general preferences
- **Groves**
  - Possible to get dominant strategy implementation with quasi-linear utilities (Efficient)
- **Clarke (or VCG)**
  - Possible to get dominant strategy implementation with quasi-linear utilities (Efficient and interim IR)
- **d'AGVA**
  - Possible to get Bayes-Nash implementation with quasi-linear utilities (Efficient, budget-balanced, ex ante IR)

Other Mechanisms

- We know what to do with
    - Voting
    - Auctions
    - Public Projects
- Are there any other “markets” that are interesting?

Bilateral Trade

- 2 agents, one buyer and one seller, each with quasi-linear utilities
- Each agent knows its own value, but not the other's
- Probability distributions are common knowledge

We want a mechanism that is

- ex post budget balanced
- ex post efficient: exchange occurs is $v_b \geq v_s$
- (interim) IR: agents have higher expected utility from participating than by not participating

Myerson-Satterthwaite Theorem

Theorem

In the bilateral trading problem no mechanism can implement an ex post budget-balanced, ex post efficient, and interim IR social choice function (even in Bayes-Nash equillibrium).

Proof

- Seller's valuation is s_L w.p. α and s_H w.p. $(1 - \alpha)$
- Buyer's valuation is b_L w.p. β and b_H w.p. $(1 - \beta)$
- Say $b_H > s_H > b_L > s_L$
- By the Revelation Principle we need only focus on truthful direct revelation mechanisms
- Let $p(b, s)$ be the probability that trade occurs given revelations b and s
  - Ex post efficiency requires: $p(b, s) = 0$ if $b = b_L$ and $s = s_H$, otherwise $p(b, s) = 1$
  - Thus $E[p|b = b_H] = 1$ and $E[p|b = b_L] = \alpha$
  - $E[p|s = s_H] = 1 - \beta$ and $E[p|s = s_L] = 1$

Proof continued

- Let $m(b, s)$ be the expected price buyer pays to the seller given revelations b and s
  - Since buyer pays what seller gets paid, this maintains budget balance ex post
  - $E[m|b] = (1-\alpha)m(b, s_H) + \alpha m(b, s_L)$
  - $E[m|s] = (1-\beta)m(b_H, s) + \beta m(b_L, s)$
- Individual rationality (IR) requires
  - $bE[p|b] - E[m|b] \geq 0$ for $b = b_L, b_H$
  - $E[m|s] - sE[p|s] \geq 0$ for $s = s_L, s_H$
- Bash-Nash incentive compatibility (IC) requires
  - $bE[p|b] - E[m|b] \geq bE[p|b'] - E[m|b']$ for all b, b'
  - $E[m|s] - sE[p|s] \geq E[m|s'] - sE[p|s']$ for all s, s'

Proof Continued

Suppose $alpha = \beta = 1/2$, $s_L = 0, s_H = y, b_L = x, b_H = x + y$ where $0 < 3x < y$

- $IR(b_L)$: $1/2x = [1/2m(b_L, s_H) + 1/2m(b_L, s_L)] \geq 0$
- $IR(s_H)$: $[1/2m(b_H, s_H) + 1/2m)b_L, s_H)] - 1/2y \geq 0$
- Summing gives $m(b_H, s_H) - m(b_L, s_L) \geq y - x$
- $IC(s_L)$: $[1/2m(b_H, s_L) + 1/2m(b_L, s_L)] \geq [1/2m(b_H, s_L) + 1/2m(b_L, s_L)]$
  - i.e.$m(b_H, s_L) - m(b_L, s_H) \geq m(b_H, s_H) - m(b_L, s_L)$
- $IC(b_H)$: $(x + y) - [1/2m(b_H, s_H) + 1/2m(b_H, s_L)] \geq 1/2(x + y) - [1/2m(b_L, s_H) + 1/2m(b_L, s_L)]$
  - i.e $x + y \geq m(b_H, s_H) - m(b_L, s_L) + m(b_H, s_L) - m(b_L, s_H)$
- So $x + y \geq 2[m(b_H, s_H) - m(b_L, s_L)] \geq 2(y - x)$ which implies $3x \geq y$. Contradiction.

Market Design Matters

- Myerson-Satterthwaite shows that under reasonable assumptions, the market will **NOT** take care of efficient allocation
- Market design does matter
  - By introducing a disinterested 3rd party (auctineer) we could get an efficient allocation