Single Item Auctions

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Outline

Auctions

- Methods for allocating goods, tasks, resources,...
- Participants
  - auctioneer
  - bidders
- Enforced agreement between auctioneer and the winning bidder(s)
- Easily implementable (e.g. over the Internet)
- Conventions
  - Auction: one seller and multiple buyers
  - Reverse auction: one buyer and multiple sellers

Todays lecture will discuss the theory in the context of auctions, but this applies to reverce auctions as well (at least in 1-item settings).

Auction Settings

- **Private value**: the value of the good depends only on the agent's own preferences
  - e.g a cake that is not resold of showed off
- **Common value:** an agent's value of an item is determined entirely by others' values (valuation of the item is identical for all agents)
  - e.g. treasury bills
- **Correlated value (interdependent value):** agent's value for an item dpends partly on its own preferences and partly on others' value for it
  - e.g. auctioning a transportation task when bidders can handle it or reauction it to others

All Pay Auction

- **Protocol:** Each bidder is free to raise their bid. When no bidder is willing to raise, the auction ends and the highest bidder wins. All bidders pay their last bid.
- **Strategy:** Series of bids as a function of agent's private value, prior estimates of others' valuations, and past bids
- **Best strategy:**

Four Common Auctions

- English auction
- First-price, sealed-bid auction
- Dutch auction
- Vickrey auction

English auction

aka first-price open-cry auction

- **Protocol:** Each bidder is free to raise their bid. When no bidder is willing to raise, the auction ends and the highest bidder wins. Highest bidder pays its last bid.
- **Strategy:** Series of bids as a function of agent's private value, prior estimates of others' valuations, and past bids
- **Best strategy:**
- **Variations:**
  - Auctioneer controls the rate of increase
  - Open-exit: Bidders have to openly declare exit with no re-entering possibilities

First-price sealed-bid auction

- **Protocol**: Each bidder submits one bid without knowing others' bids. The highest bidder wins the item at the price of it's bid
- **Strategy:** Bid as a function of agent's private value and its prior estimates of others' valuations
- **Best strategy:**

Example

Assume there are 2 agents (1 and 2) with values v_1, v_2 drawn uniformly from $[0, 1]$. Utility of agent i if it bids b_i and wins is $u_i = v_i - b_i$.
Assume that agent 2's bidding strategy is $b_2(v_2) = v_2/2$. How should 1 bid? (i.e. what is $b(v_1) = z$?).

$$U_1 = \int_{z=0}^{2z} (v_1 - z)dz = (v_1 - z)2z = 2zv_1 - 2z^2$$

Note: given $z = b_2(v_2) = v_2/2$, 1 only wins if $v_2 < 2z$
Therefore,

$$\arg\max_z [2zv_1 - 2z^2] = v_1/2$$

Similar arguement for agent 2, assuming $b_1(v_1) = v_1/2$.

Example

Assume that there are 2 risk-neutral bidders, 1 and 2.

- Agent 1 knows that 2's value is 0 or 100 with equal probability
- 1's value of 400 is common knowledge

What is a Nash equilibrium?

Dutch auction

Descending auction

- **Protocol:** Auctioneer continuously lowers the price until a bidder takes the item at the current price
- **Strategy:** Bid as a function of agent's private value and prior estimates of others' valuations
- **Best strategy:**
- Dutch flower market, Ontario tobacco auctions, Filene's basement,...

Dutch (Aalsmeer) flower auction

Vickrey Auction

aka Second price, sealed bid auction

- **Protocol:** Each bidder submits one bid without knowing the others' bids. The highest bidder wins and pays an amount equal to the second highest bid.
- **Strategy:** Bid as a function of agent's private value and its prior estimates of others' valuations.
- **Best strategy:**
- Widely advocated for computational multiagent systems
- Old (Vickrey 1961) but not widely used by humans

Vickrey auction

The Vickrey auction is a special case of the Clarke Tax.

- Who pays?
  - The bidder who takes the item away from the others (making the others worse off)
  - Others pay nothing
- How much does the winner pay?
  - The declared value that the good would have had for the others had the winner stayed home (second highest bid)

Results for Private Value Auctions

- Dutch and first-price sealed-bid auctions are strategically equivalent
- For risk neutral agents, Vickrey and English auctions are strategically equivalent
  - Dominant strategies
- All four auctions allocate item efficiently
  - Assuming no reservation price for the auctioneer

Revenue

Theorem (Revenue Equivalence)

Suppose that

- *values are independently and identically distributed and*
- *all bidders are risk neutral.*

Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue.

Revenue equivalence fails to hold if agents are not risk neutral.

- Risk averse bidders: Dutch, first-price $\geq$ Vickrey, English
- Risk seeking bidders: Dutch, first-price $\leq$ Vickrey, English

Optimal Auctions

Common Value Auctions

In a common value auction, the item has some unknown value and each agent has some partial information about the value. Each agent i has signal $X_i \in [0, \omega_i]$. The value V of the item is

$$V = v(X_1, \ldots, X_n)$$

- Examples
  - Art auctions and resale
  - Construction companies effected by common events (e.g. weather)
  - Oil drilling

Common Value Auctions

- At time of bidding the common value is unknown
- Bidders may have imperfect estimates about the value
- True value only observed after the auction has taken place

Winner’s Curse

- No agent knows for sure the true value of the item
- The winner is the agent who made the highest guess
- If bidders all had “reasonable” information about the value, then the average of all guesses should be correct
  - i.e. the winner has overbid!

Agents should shade their bids downward (even in English and Vicrey auctions).

Results for Non-Private Value Auctions

- Dutch and first-price sealed-bid are strategically equivalent
- Vickrey and English are not strategically equivalent
- All four auctions are efficient

Theorem (Revenue Non-Equivalence)

With more than 2 bidders, the expected revenues are not the same:

$$\textit{English} \geq \textit{Vickrey} \geq \textit{Dutch} = \textit{first-price sealed-bid}$$

Bidder Collusion

Example: $v_1 = 20$ and $v_i = 18$ for other bidders.

- Collusive agreement for English auction: 1 bids 6 and others bid 5. This is self-enforcing
- Collusive agreement for Vickrey auction: 1 bids 20 and others bid 5. This is self-enforcing
- In first-price or Dutch auction, if 1 bids below 18, others are motivated to break the collusion
- Need to identify coalition parties

Misbehaving Auctioneers

- Shill bidding is bidding to artifically increase an item's price.
  - In theory, only a problem in non-private value auctions
  - English and all-pay auctions are vulnerable
    - Classic analysis ignores the possibility of shills
  - Vickrey, first-price, and Dutch are not vulnerable
- In Vickrey auction, auctioneer can overstate 2nd highest bid
- Auctioneer can refuse to sell once the auction has closed

Undesirable Information Revelation

- Vickrey and English auctions reveal agents' strategic marginal cost information since truthful bidding is a dominant strategy
  - Observed problems with subcontractors
- First-price and Dutch may not reveal this information as accurately
  - No dominant strategy and bidding decisions depend on beliefs of others

Sniping

Sniping is bidding very late in the auction in the hopes that other bidders do not have time to respond. This is a real issue in online auctions.

	Hypotheses	Predicted contribution to late bidding
Strategic hypotheses	• *Rational response to naïve English auction behavior or to shill bidders:* bidders bid late to avoid bidding wars with incremental bidders. • *Collusive equilibrium:* bidders bid late to avoid bidding wars with other like-minded bidders. • *Informed bidders protecting their information:* e.g. late bidding by experts/dealers.	All three strategic hypotheses suggest more late bidding on eBay than on Amazon, with a bigger effect for more experienced bidders. Plus (via the third point) more late bidding in categories in which expertise is important than in categories in which it is not.
Non-strategic hypotheses	Bidders bid late because … • of procrastination; • search engines present soon-to-expire auctions first; • of a desire to retain flexibility to bid on other auctions offering the same item; • they remain unaware of the proxy bidding system; • of an increase in the willingness to pay over time caused by, e.g., an endowment effect; or because • bidders don't like to leave bids "hanging."	No difference between eBay and Amazon.

Sniping

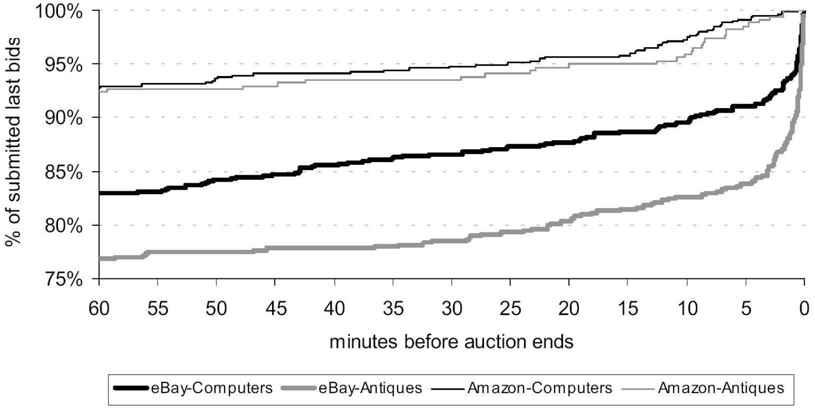

Figure 1a–Cumulative distributions over time of bidders' last bids

Sniping

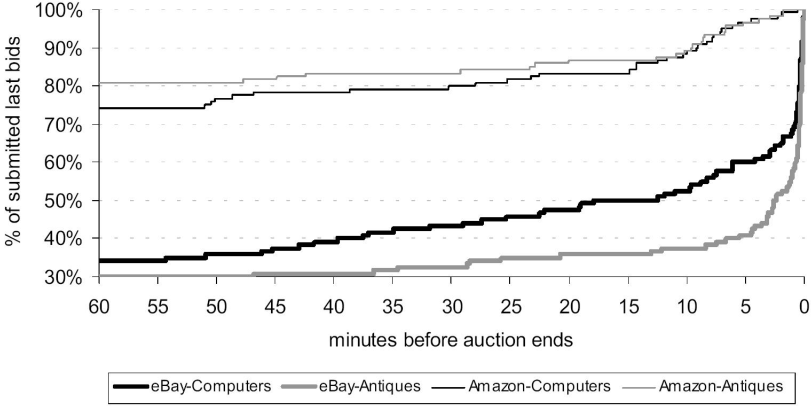

Figure 1b–Cumulative distributions over time of auctions' last bids

Summary

- Auctions are nontrivial but often analyzable
  - Important to understand merits and limitations
  - Unintuitive auctions may have better properties (i.e. Vickrey auction)
- Choice of a good auction depends on the setting in which the protocol is used