CS 886: Game-theoretic Methods for Computer Science
Assignment 1

September 24, 2006

Due Wednesday, October 4 in class. In this assignment you are expected to work individually. You may use any sources that you want, but you must cite them. My office hours are Mondays 1:30-2:30 in DC 2518. You can also email me if you have questions.

1. (10 points) Throughout this question, you may restrict your analysis to pure strategies.

   (a) Draw the normal form game of the game tree in Figure 1.

   (b) Name the dominant strategy equilibria, if there are any.

   (c) Name the Nash equilibria of this game, if there are any.

   (d) Name the subgame perfect Nash equilibria in the game, if there are any.

   (e) Name the Pareto efficient outcomes of this game, if there are any.

   (f) Name the social welfare maximizing outcomes of this game, if there are any.

2. *Game of Chicken* Two teenagers play the following risky game. They drive towards each other at stop speed in separate cars. Just before collision each one has the choice of continuing straight or avoiding collision by turning right. If both continue straight then they both die. If one continues straight while the other turns they both live, but the one who went straight gets boasting rights and the is humiliated. If both turn, then both survive and both are moderately humiliated. The game is represented in the table in Figure 2.

   (a) (5 pts) Does this game have pure strategy Nash equilibria? If so, what are they?

   (b) (13 pts) What are the mixed strategy Nash equilibria of this game?

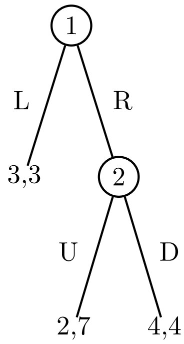

Figure 1:

	straight	turn
straight	-3,-3	2,0
turn	0,2	1,1

Figure 2:

(c) (2 pts) In each equilibrium, what is the probability that the teenagers will die?

3. (10 pts) Agents 1 and 2 play Split-the-Dollar. Each agent simultaneously name shares that they want, where s_1 is agent 1's requested share and s_2 is agent 2's requested share. If $s_1 + s_2 \leq 1$ then both agent gets their requested share. If $s_1 + s_2 > 1$ then both agents get zero. What are the pure strategy Nash equilibria in this game?

4. An agent's strategy is *strictly dominated* if that agent has another strategy that gives strictly better payoff to the agent no matter what strategies other agents do. An agent's strategy is *weakly dominated* if that agent has another strategy that gives at least equally high payoff to the agent no matter what other agents do, and strictly higher payoff to the agent for at least one choice of strategies of by the others. To solve a game, we can iteratively eliminate dominated strategies until all remaining strategies are undominated.

   (a) (10 pts) Prove that if strategies $s^* = (s_1^*, \ldots, s_n^*)$ are a Nash equilibrium in a normal form game, then they survive iterated elimination of strictly dominated strategies. (Hint: By contradiction, assume that one of the strategies in the Nash equilibrium is eliminated by iterated elimination of dominated strategies).

   (b) (10 pts) Prove that if the process of iterated elimination of strictly dominated strategies results in a unique strategy profile $s^* = (s_1^*, \ldots,$

s_n^*) then this is a Nash equilibrium of the game. (Hint: By contradiction, assume there exists some agent i for which $s_i \neq s_i^*$ is preferred over s_i^*, and show a contradiction with the fact that s_i was eliminated.)