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On Keys and Functional Dependencies as First-Class Citizens in Description Logics

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Abstract We investigate whether *identification constraints* such as keys and functional dependencies can be granted full status as concept constructors in a Boolean-complete description logic. In particular, we show that surprisingly simple forms of such constraints lead to undecidability of the associated logical implication problem if they are allowed within the scope of a negation or on the left-hand side of inclusion dependencies. We then show that allowing a very general form of identification constraint to occur in the scope of *monotone* concept constructors on the right-hand side of inclusion dependencies still leads to decidable implication problems. We consider the relationship between certain classes of identification constraints and nominals.

Keywords Description logics · Path-functional dependency · Relational keys

1 Introduction

To date, description logics (DLs) have incorporated keys or functional dependencies in one of two ways. The first adds a separate family of terminological constraints to inclusion dependencies, for example, in the form of a *key box* [5, 7, 13, 14], while the second avoids this by adding a new concept constructor called a *path-functional dependency* (PFD) [11, 19, 20]. However, the second approach still falls short of a full integration of keys or functional dependencies because of syntactic restrictions on occurrences of PFDs and on the syntax of the PFD constructor itself. In particular, all earlier work has required that any occurrence of this constructor appear only at the

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top level on the right-hand side of inclusion dependencies and that the left-hand sides of PFDs themselves are nonempty. Note that an ordinary functional dependency of the form "$\{\} \rightarrow A$" has an empty left-hand side and consequently enforces a fixed A value. In this paper, we investigate whether such syntactic restrictions are necessary—unfortunately, it turns out that this is indeed the case—and study the limits of decidability in such a setting. The main contributions are as follows.

- We show that allowing PFDs on the left-hand side of inclusion dependencies or in the scope of a containing negation on the right-hand side leads to undecidability. Notably, this remains true when PFDs are limited to very simple forms of relational keys or functional dependencies.
- Conversely, we show that allowing PFDs within the scope of *monotone* concept constructors on the right-hand side of inclusion dependencies still leads to decidable implication problems.
- We show that allowing left-hand sides of PFDs to be empty also leads to undecidability. Specifically, we first show that the introduction of an ABox to previously decidable PFD dialects already makes logical implication undecidable. The result follows by showing that such PFDs can simulate nominals.

DLs have become an important part of the Semantic Web. Indeed, OWL—the current standard for capturing Semantic Web ontologies—is based largely on a DL dialect. DLs have also been used as a lingua franca for a large variety of languages for capturing metadata: UML class diagrams, ER models, relational schema, object-oriented schema, DTDs for XML, and XML itself are all examples [6, 15].

1.1 Identification is Important

Identification constraints are fundamentally tied to issues of equality; and as Web services with query languages such as SWRL that are based on OWL are introduced, important questions inevitably surface when finding execution strategies for services and when communicating results of such services. For example, *How can one reliably identify resources?* and *Is there at most one kind of Web service?* With the addition of the PFD concept constructor along the lines considered in this paper, it becomes possible to express, for example, that *among all possible clients, social security numbers are a reliable way of identifying those that are registered.* In particular, this information can be captured by the following inclusion dependency.

$$\text{Client} \sqsubseteq \neg\text{Registered} \sqcup \text{Client} : \mathit{SIN} \rightarrow \mathit{Id}$$

To paraphrase: *If a client is registered, then no other client will share his social insurance number.* Note that social insurance numbers may not be a reliable way of identifying an arbitrary unregistered client in general.

A number of additional applications and capabilities become possible after removing syntactic restrictions on PFDs, beyond the fact that a simple and elegant

presentation of the associated DL would ensue. For example, to say that *all information about clients is located at a single site*, one can add the following dependency.

$$\text{Client} \sqsubseteq \text{Client} :\rightarrow \mathit{LocationOfData}$$

Again to paraphrase: *For any pair of clients, both will agree on the location of available data.*

As we shall see, relaxing existing restrictions to accommodate the first example is possible because disjunction is a monotone concept constructor, but it is not possible for the second example without the introduction of alternative restrictions on the use of PFDs or syntax of the PFD constructor itself.

1.2 Background and Related Work

PFDs were first introduced and studied in the context of object-oriented data models [9, 23]. An FD concept constructor was subsequently proposed and incorporated in Classic [4], an early DL with a PTIME reasoning procedure, without changing the complexity of its implication problem. The generalization of this constructor to PFDs alone leads to EXPTIME completeness of the implication problem [11]; this complexity remains unchanged in the presence of additional concept constructors common in rich DLs [19, 20]. Note that all earlier work has assumed the above syntactic restrictions on occurrences of the PFD concept constructor in inclusion dependencies.

Calvanese et al. have considered a DL with functional dependencies and a general form of keys added as additional varieties of dependencies, called a *key box* [5]. They show that their dialect is undecidable for DLs with inverse roles but becomes decidable when unary functional dependencies are disallowed. This line of investigation is continued in the context of PFDs and inverse attributes, with analogous results [18]. (We therefore disallow inverse attributes in this paper to exclude an already known cause for undecidability.)

A form of key dependency with left-hand-side feature paths has been considered for a DL coupled with various concrete domains [13, 14]. In this case, the authors explore how the complexity of satisfaction is influenced by the selection of a concrete domain together with various syntactic restrictions on the key dependencies themselves.

PFDs have been used in a number of applications: in object-oriented schema diagnosis and synthesis [2, 3], in query optimization [8, 10], and in the selection of indexing for a database [16].

We note that the results reported in this paper are an expansion of earlier preliminary work in Toman and Weddell [21].

The remainder of the paper is organized as follows. The definition of $\mathcal{DLFD}$, a Boolean complete DL based on attributes that includes the PFD concept constructor, immediately follows. In Section 3, we show that the interaction of this constructor with negation leads to undecidability for a variety of simple cases of PFDs. Section 4 then shows how decidability can be regained while still allowing PFDs in the scope of monotone concept constructors on the right-hand sides of inclusion dependencies, most significantly in the scope of concept union and attribute restriction. In Section 5,

we consider relaxing the requirement that PFDs have nonempty left-hand sides, showing that both this and a (weaker) alternative of admitting an ABox leads to undecidability. Our summary comments follow in Section 6.

2 Definitions

Our investigations are based on the following dialect of description logic called $\mathcal{DLFD}$. To simplify the presentation, we base the dialect on *attributes* (also called *features*) instead of the more common case of roles. Note that $\mathcal{ALCN}$ with a suitable PFD construct can simulate our dialect. Conversely, $\mathcal{DLFD}$ can simulate $\mathcal{ALCQI}$ [17, 19].

Definition 1 (Description Logic $\mathcal{DLFD}$) Let F and C be sets of (names of) attributes and primitive concepts, respectively. A *path expression* is defined by the grammar "$\mathsf{Pf} ::= f.\mathsf{Pf} \mid Id$" for $f \in \mathsf{F}$. We define derived *concept descriptions* by the grammar on the left-hand side of Fig. 1. A concept description obtained by using the fourth production of this grammar is called an *attribute value restriction*. A concept description obtained by using the final production is called a *path functional dependency* (PFD). Note that we assume for this production that $k > 0$, that the left-hand side of a PFD is nonempty. In addition, a PFD is called. (1) *unary* when $k = 1$, (2) *key* when the right-hand side is Id, and (3) *simple* when there is no path expression with more than a single attribute name.

An *inclusion dependency* $\mathcal{C}$ is an expression of the form $D \sqsubseteq E$. A *terminology* $\mathcal{T}$ consists of a finite set of inclusion dependencies.

The *semantics* of expressions is defined with respect to a structure $(\Delta, \cdot^{\mathcal{I}})$, where Δ is a domain of "objects" and $(.)^{\mathcal{I}}$ an interpretation function that fixes the interpretation of primitive concepts C to be subsets of Δ and primitive attributes f to be total functions $(f)^{\mathcal{I}} : \Delta \to \Delta$. The interpretation is extended to path expressions, $(Id)^{\mathcal{I}} = \lambda x.x$, $(f.\mathsf{Pf})^{\mathcal{I}} = (\mathsf{Pf})^{\mathcal{I}} \circ (f)^{\mathcal{I}}$ and derived concept descriptions D and E as defined on the right-hand-side of Fig. 1.

An interpretation *satisfies an inclusion dependency* $D \sqsubseteq E$ if $(D)^{\mathcal{I}} \subseteq (E)^{\mathcal{I}}$. The *logical implication problem* asks whether $\mathcal{T} \models D \sqsubseteq E$ holds; that is, for a *posed question* $D \sqsubseteq E$, if $(D)^{\mathcal{I}} \subseteq (E)^{\mathcal{I}}$ for all interpretations that satisfy all inclusion dependencies in $\mathcal{T}$.

Syntax	Semantics: Defn of "$(\cdot)^{\mathcal{I}}$"
$D, E ::= C$	$(C)^{\mathcal{I}} \subseteq \Delta$
$\mid \; D_1 \sqcap D_2$	$(D_1)^{\mathcal{I}} \cap (D_2)^{\mathcal{I}}$
$\mid \; D_1 \sqcup D_2$	$(D_1)^{\mathcal{I}} \cup (D_2)^{\mathcal{I}}$
$\mid \; \forall f.D$	$\{x : (f)^{\mathcal{I}}(x) \in (D)^{\mathcal{I}}\}$
$\mid \; \neg D$	$\Delta \setminus (D)^{\mathcal{I}}$
$\mid \; D : \mathsf{Pf}_1, ..., \mathsf{Pf}_k \to \mathsf{Pf}$	$\{x : \forall\, y \in (D)^{\mathcal{I}}.$ $\bigwedge_{i=1}^{k} (\mathsf{Pf}_i)^{\mathcal{I}}(x) = (\mathsf{Pf}_i)^{\mathcal{I}}(y) \Rightarrow (\mathsf{Pf})^{\mathcal{I}}(x) = (\mathsf{Pf})^{\mathcal{I}}(y)\}$

Fig. 1 Syntax and semantics of $\mathcal{DLFD}$

To improve readability in the remainder of the paper, we follow the simple protocol of identifying single attributes f with path expressions $f.Id$.

3 Undecidability

Allowing arbitrary use of very simple varieties of the PFD concept constructor leads to undecidable implication problems, particularly for three *boundary* cases:

1. when all PFDs are simple and key,
2. when all PFDs are simple and unary, and
3. when all PFDs are simple and nonkey.

In the first case, a PFD has the form $C : f_1, \ldots, f_k \rightarrow Id$, which captures the standard notion of *relational keys*. In the second case, a PFD has either the form $C : f \rightarrow g$ or the form $C : f \rightarrow Id$. The standard notion of a (relational) *functional dependency* (FD) is captured by the third case, in which a PFD resembles $C : f_1, \ldots, f_k \rightarrow f$.

The three cases are exhaustive in the sense that the only possibility not covered happens when all PFDs have the form $C : f \rightarrow Id$ (i.e., are simple, unary, and key). However, it is a straightforward exercise in this situation to map logical implication problems to alternative formulations in decidable DL dialects with inverses and number restrictions. In the rest of this section, we elaborate on each of these cases. Notably, the reductions make no use of attribute value restrictions in the first two of these cases; they rely solely on PFDs and the standard Boolean constructors.

The undecidability results are based on a reduction of the unrestricted tiling problem to the $\mathcal{DLFD}$ implication problem. An instance U of this problem is a triple (T, H, V) where T is a finite set of tile types and $H, V \subseteq T \times T$ two binary relations. A *solution* to T is a mapping $t : \mathbf{N} \times \mathbf{N} \rightarrow T$ such that $(t(i, j), t(i+1, j)) \in H$ and $(t(i, j), t(i, j+1)) \in V$ for all $i, j \in \mathbf{N}$. This problem is Π^0_0-complete [1, 22].

3.1 PFDs that are Simple and Key (Relational Keys)

The reduction constructs a terminology for a given tiling problem $U = (T, H, V)$, denoted $\mathcal{T}^1_U$, by first establishing an *integer grid*. This is achieved in three steps.

1. Introduce primitive concepts A, B, C, and D to serve as possible grid points.

$$\mathrm{A} \sqcap \mathrm{B} \sqsubseteq \bot \quad \mathrm{A} \sqcap \mathrm{C} \sqsubseteq \bot \quad \mathrm{A} \sqcap \mathrm{D} \sqsubseteq \bot \quad \mathrm{B} \sqcap \mathrm{C} \sqsubseteq \bot \quad \mathrm{B} \sqcap \mathrm{D} \sqsubseteq \bot \quad \mathrm{C} \sqcap \mathrm{D} \sqsubseteq \bot$$

2. Create an "infinitely branching" tree of squares. Such a tree can be rooted at a hypothetical top-left with, for example, an A object.

$$\begin{aligned}
\mathrm{A} &\sqsubseteq \neg(\mathrm{B} : g, k \rightarrow Id) \sqcap \neg(\mathrm{C} : f, g \rightarrow Id) \\
\mathrm{B} &\sqsubseteq \neg(\mathrm{A} : f, h \rightarrow Id) \sqcap \neg(\mathrm{D} : f, g \rightarrow Id) \\
\mathrm{C} &\sqsubseteq \neg(\mathrm{A} : h, k \rightarrow Id) \sqcap \neg(\mathrm{D} : g, k \rightarrow Id) \\
\mathrm{D} &\sqsubseteq \neg(\mathrm{B} : h, k \rightarrow Id) \sqcap \neg(\mathrm{C} : f, h \rightarrow Id)
\end{aligned}$$

3. Flatten and align the tree into an integer grid using keys.

$$\mathrm{A} \sqsubseteq \mathrm{A} : h \rightarrow Id \quad \mathrm{B} \sqsubseteq \mathrm{B} : k \rightarrow Id \quad \mathrm{C} \sqsubseteq \mathrm{C} : f \rightarrow Id \quad \mathrm{D} \sqsubseteq \mathrm{D} : g \rightarrow Id$$

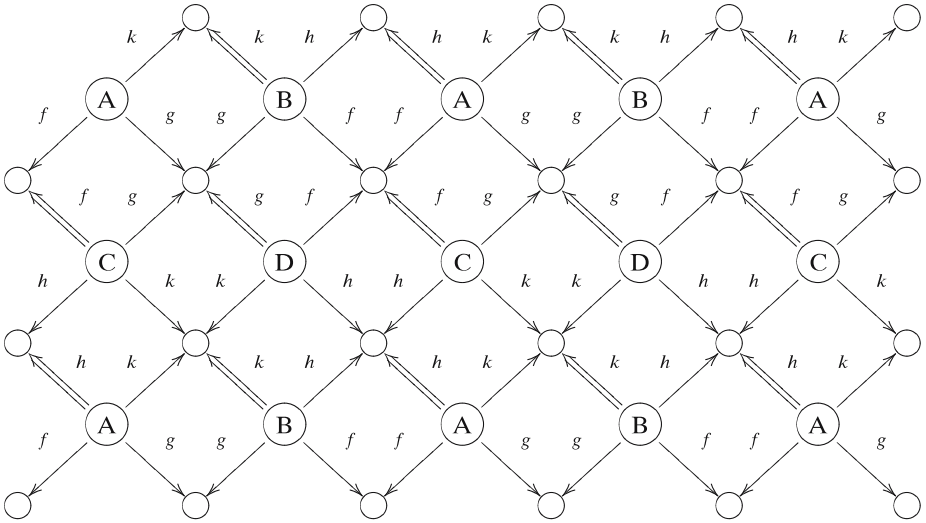

Fig. 2 Defining a grid (first and second case)

The accumulated effect of these inclusion dependencies on an interpretation is illustrated in Fig. 2. Note that the *thick edges* indicate places where flattening of the infinitely branching tree of squares happens.

We model the tiling problem U using primitive concepts T_i for each tile type $t_i \in T$, asserting that $\mathrm{T}_i \sqcap \mathrm{T}_j \sqsubseteq \bot$ for all $i < j$. The tiles are placed on the grid points with the inclusion dependency $(\mathrm{A} \sqcup \mathrm{B} \sqcup \mathrm{C} \sqcup \mathrm{D}) \sqsubseteq \bigsqcup_{t_i \in T} \mathrm{T}_i$. The adjacency rules for the instance U of the tiling problem can now be captured as follows:

– for $(t_i, t_j) \notin H$:

$$\begin{array}{ll} \mathrm{A} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{B} \sqcap \mathrm{T}_j) : g \to Id & \mathrm{B} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{A} \sqcap \mathrm{T}_j) : f \to Id \\ \mathrm{C} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{D} \sqcap \mathrm{T}_j) : k \to Id & \mathrm{D} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{C} \sqcap \mathrm{T}_j) : h \to Id \end{array}$$

– for $(t_i, t_j) \notin V$:

$$\begin{array}{ll} \mathrm{A} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{C} \sqcap \mathrm{T}_j) : f \to Id & \mathrm{C} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{A} \sqcap \mathrm{T}_j) : h \to Id \\ \mathrm{B} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{D} \sqcap \mathrm{T}_j) : g \to Id & \mathrm{D} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{B} \sqcap \mathrm{T}_j) : k \to Id \end{array}$$

where T_i corresponds to tile type $t_i \in T$. The above inclusion dependencies form a terminology $\mathcal{T}_U^1$ associated with an unrestricted tiling problem U that immediately yields the following result.

Theorem 2 *An instance $U = (T, H, V)$ of the infinite tiling problem admits a solution if and only if*

$$\mathcal{T}_U^1 \not\models \mathrm{A} \sqsubseteq \bot.$$

Corollary 3 *The logical implication problem for $\mathcal{DLFD}$ with PFDs that are simple and key is undecidable. This remains true in the absence of attribute values restrictions.*

3.2 PFDs that are Simple and Unary

Again, the reduction constructs a terminology for a given tiling problem $U = (T, H, V)$, denoted this time as $\mathcal{T}_U^2$, by first establishing an integer grid. For this case, an extra step is needed.

1. As before, introduce primitive concepts A, B, C, and D to serve as possible grid points.

$$\mathrm{A} \sqcap \mathrm{B} \sqsubseteq \bot \quad \mathrm{A} \sqcap \mathrm{C} \sqsubseteq \bot \quad \mathrm{A} \sqcap \mathrm{D} \sqsubseteq \bot \quad \mathrm{B} \sqcap \mathrm{C} \sqsubseteq \bot \quad \mathrm{B} \sqcap \mathrm{D} \sqsubseteq \bot \quad \mathrm{C} \sqcap \mathrm{D} \sqsubseteq \bot$$

2. To create an analogous infinitely branching tree of squares, first create "raw material" consisting of the necessary chains.

$$\begin{array}{ll} \mathrm{A} \sqsubseteq \neg(\mathrm{B} : k \to Id) \sqcap \neg(\mathrm{C} : f \to Id) & \mathrm{B} \sqsubseteq \neg(\mathrm{A} : h \to Id) \sqcap \neg(\mathrm{D} : g \to Id) \\ \mathrm{C} \sqsubseteq \neg(\mathrm{A} : h \to Id) \sqcap \neg(\mathrm{D} : g \to Id) & \mathrm{D} \sqsubseteq \neg(\mathrm{B} : k \to Id) \sqcap \neg(\mathrm{C} : f \to Id) \end{array}$$

3. Shape the chains into squares with functional dependencies.

$$\begin{array}{ll} \mathrm{A} \sqsubseteq (\mathrm{B} : k \to g) \sqcap (\mathrm{C} : f \to g) & \mathrm{B} \sqsubseteq (\mathrm{A} : h \to f) \sqcap (\mathrm{D} : g \to f) \\ \mathrm{C} \sqsubseteq (\mathrm{A} : h \to k) \sqcap (\mathrm{D} : g \to k) & \mathrm{D} \sqsubseteq (\mathrm{B} : k \to h) \sqcap (\mathrm{C} : f \to h) \end{array}$$

4. Flatten and align the tree, using keys, to obtain an integer grid.

$$\mathrm{A} \sqsubseteq \mathrm{A} : h \to Id \quad \mathrm{B} \sqsubseteq \mathrm{B} : k \to Id \quad \mathrm{C} \sqsubseteq \mathrm{C} : f \to Id \quad \mathrm{D} \sqsubseteq \mathrm{D} : g \to Id$$

The accumulated effect of these inclusion dependencies on an interpretation is the same as in Fig. 2. Since a tiling problem can then be overlaid on this grid as we did for PFDs that are simple and key, we have the following result.

Theorem 4 *An instance $U = (T, H, V)$ of the infinite tiling problem admits a solution if and only if*

$$\mathcal{T}_U^2 \not\models \mathrm{A} \sqsubseteq \bot.$$

Corollary 5 *The logical implication problem for $\mathcal{DLFD}$ with PFDs that are simple and unary is undecidable. This remains true in the absence of attribute value restrictions.*

3.3 PFDs that are Simple and NonKey (Relational FDs)

The final reduction also constructs a terminology for a given tiling problem $U = (T, H, V)$, denoted $\mathcal{T}_U^3$, by first establishing an integer grid in three steps.

1. Introduce primitive disjoint concepts A, B, C, and D. Here, however, they will serve as "grid point interfaces." (The actual grid points will eventually align with values of their attributes.)

$$\mathrm{A} \sqcap \mathrm{B} \sqsubseteq \bot \quad \mathrm{A} \sqcap \mathrm{C} \sqsubseteq \bot \quad \mathrm{A} \sqcap \mathrm{D} \sqsubseteq \bot \quad \mathrm{B} \sqcap \mathrm{C} \sqsubseteq \bot \quad \mathrm{B} \sqcap \mathrm{D} \sqsubseteq \bot \quad \mathrm{C} \sqcap \mathrm{D} \sqsubseteq \bot$$

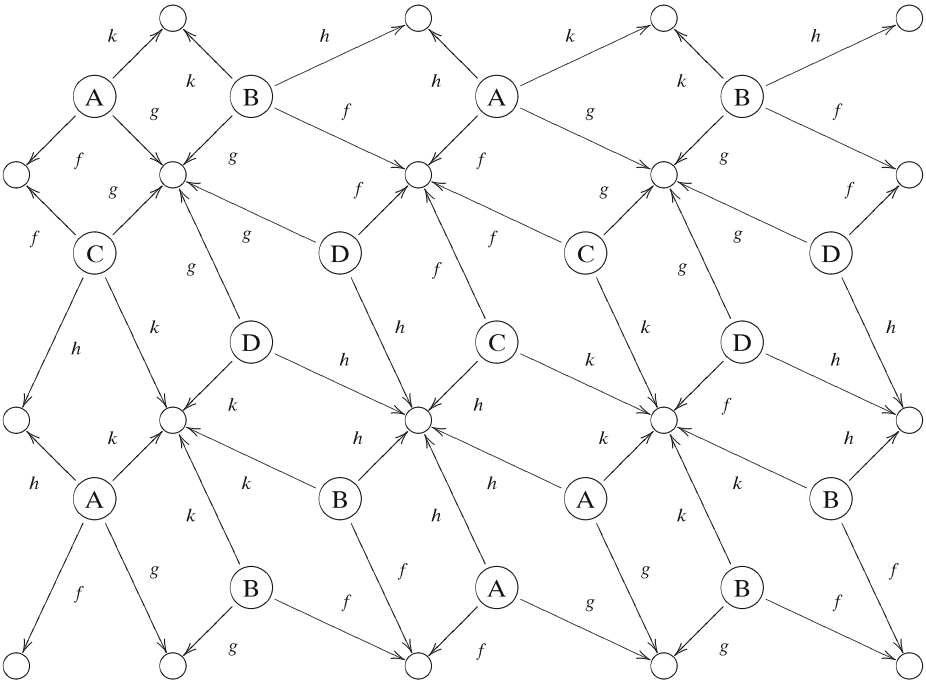

Fig. 3 Defining a grid (final third case)

2. To (eventually) establish a grid, create an infinitely branching tree of squares in a fashion analogous to the previous cases. Again, such a tree can be rooted at a hypothetical top-left with an A object.

$$\begin{aligned}
\mathrm{A} &\sqsubseteq \neg(\mathrm{B} : g, k \to z) \sqcap \neg(\mathrm{C} : f, g \to z) \\
\mathrm{B} &\sqsubseteq \neg(\mathrm{A} : f, h \to z) \sqcap \neg(\mathrm{D} : f, g \to z) \\
\mathrm{C} &\sqsubseteq \neg(\mathrm{A} : h, k \to z) \sqcap \neg(\mathrm{D} : g, k \to z) \\
\mathrm{D} &\sqsubseteq \neg(\mathrm{B} : h, k \to z) \sqcap \neg(\mathrm{C} : f, h \to z)
\end{aligned}$$

   In this construction, the attribute z is arbitrary and is employed simply to satisfy our requirement that PFDs are simple and nonkey. Note that, for example, any given A object must continue to have a distinct B object to its right and a distinct C object below (according to the first of these inclusion dependencies).
3. Flatten the following equivalent to (unary) relational FDs, and align the tree into the desired integer grid.

$$\mathrm{A} \sqsubseteq \mathrm{A} : h \to g \quad \mathrm{B} \sqsubseteq \mathrm{B} : k \to f \quad \mathrm{C} \sqsubseteq \mathrm{C} : f \to k \quad \mathrm{D} \sqsubseteq \mathrm{D} : g \to h$$

The accumulated effect of these inclusion dependencies on an interpretation is illustrated in Fig. 3. In this case, the grid points correspond to the small circles, which in turn correspond to attribute values of *grid point interface objects* with labels A, B, C, and D. Note that a harmless consequence of the above is that such interface objects come in pairs ("top" and "left" boundary cases excepted).

Again, we model the tiling problem U using primitive concepts T_i for each tile type $t_i \in T$, asserting that $\mathrm{T}_i \sqcap \mathrm{T}_j \sqsubseteq \bot$ for all $i < j$. The tiles are now placed on the grid points with the following inclusion dependency.

$$(\mathrm{A} \sqcup \mathrm{B} \sqcup \mathrm{C} \sqcup \mathrm{D}) \sqsubseteq \forall f.\Big(\bigsqcup_{t_i \in T} \mathrm{T}_i\Big) \sqcap \forall g.\Big(\bigsqcup_{t_i \in T} \mathrm{T}_i\Big) \sqcap \forall h.\Big(\bigsqcup_{t_i \in T} \mathrm{T}_i\Big) \sqcap \forall k.\Big(\bigsqcup_{t_i \in T} \mathrm{T}_i\Big).$$

As before, a final collection of inclusion dependencies is needed to capture the adjacency rules for the instance U of the tiling problem:

– for $(t_i, t_j) \notin H$:

$$(\mathrm{A} \sqcap \forall f.\mathrm{T}_i \sqcap \forall g.\mathrm{T}_j) \sqsubseteq \bot \qquad (\mathrm{B} \sqcap \forall g.\mathrm{T}_i \sqcap \forall f.\mathrm{T}_j) \sqsubseteq \bot$$
$$(\mathrm{C} \sqcap \forall h.\mathrm{T}_i \sqcap \forall k.\mathrm{T}_j) \sqsubseteq \bot \qquad (\mathrm{D} \sqcap \forall k.\mathrm{T}_i \sqcap \forall h.\mathrm{T}_j) \sqsubseteq \bot$$

– for $(t_i, t_j) \notin V$:

$$(\mathrm{A} \sqcap \forall h.\mathrm{T}_i \sqcap \forall f.\mathrm{T}_j) \sqsubseteq \bot \qquad (\mathrm{B} \sqcap \forall k.\mathrm{T}_i \sqcap \forall g.\mathrm{T}_j) \sqsubseteq \bot$$
$$(\mathrm{C} \sqcap \forall f.\mathrm{T}_i \sqcap \forall h.\mathrm{T}_j) \sqsubseteq \bot \qquad (\mathrm{D} \sqcap \forall g.\mathrm{T}_i \sqcap \forall k.\mathrm{T}_j) \sqsubseteq \bot$$

where T_i corresponds to tile type $t_i \in T$. The above inclusion dependencies form a terminology $\mathcal{T}_U^3$ associated with an unrestricted tiling problem U that immediately yields the following for our final boundary case.

Theorem 6 *An instance $U = (T, H, V)$ of the infinite tiling problem admits a solution if and only if*

$$\mathcal{T}_U^3 \not\models \mathrm{A} \sqsubseteq \bot.$$

Corollary 7 *The logical implication problem for $\mathcal{DLFD}$ with PFDs that are simple and nonkey is undecidable.*

4 On Regaining Decidability

We now show that undecidability is a consequence of allowing PFDs to occur within the scope of negation. In particular, and for the remainder of the paper, we shall assume a *limited* $\mathcal{DLFD}$ in which inclusion dependencies, $\mathrm{D} \sqsubseteq \mathrm{E}$, are presumed to adhere to the following less general grammar.

$$D ::= C \mid D_1 \sqcap D_2 \mid D_1 \sqcup D_2 \mid \forall f.D \mid \neg D$$
$$E ::= D \mid E_1 \sqcap E_2 \mid E_1 \sqcup E_2 \mid \forall f.E \mid D : \mathsf{Pf}_1, ..., \mathsf{Pf}_k \rightarrow \mathsf{Pf}$$

Observe that PFDs must now occur on right-hand sides of inclusion dependencies either at the top level or *within the scope of monotone concept constructors*; this condition implies that limited $\mathcal{DLFD}$ is a strict generalization of earlier dialects. Note that allowing PFDs on left-hand sides is equivalent to allowing PFDs in the scope of negation.

Example 8 $\mathrm{D}_1 \sqsubseteq \neg(\mathrm{D}_2 : f \rightarrow g)$ is equivalent to $\mathrm{D}_1 \sqcap (\mathrm{D}_2 : f \rightarrow g) \sqsubseteq \bot$.

In the following, we reduce logical implication problems in limited $\mathcal{DLFD}$ to simpler formulations for which existing decisions procedures can be applied [17, 19].

4.1 Transformation of Terminologies

We start by showing that allowing PFDs in *monotone* concept constructors within terminologies can be avoided by a syntactic transformation.

Definition 9 (Simple Constraints and Terminologies) An inclusion dependency $\mathrm{D} \sqsubseteq \mathrm{E} \in \mathcal{T}$ is called *simple* if it conforms to limited $\mathcal{DLFD}$ and if the right-hand side can be parsed by the following grammar.

$$E ::= D \mid D : \mathsf{Pf}_1, ..., \mathsf{Pf}_k \rightarrow \mathsf{Pf}$$

A terminology $\mathcal{T}$ is called *simple* if all its inclusion dependencies are simple.

For a given terminology $\mathcal{T}$, we construct a simple terminology $\mathcal{T}^{\text{simp}}$ by rewriting the right-hand sides of inclusion dependencies as follows:

$$\begin{aligned}
(\mathrm{D} \sqsubseteq \mathrm{D}')^{\text{simp}} &= \{\mathrm{D} \sqsubseteq \mathrm{D}'\} \\
(\mathrm{D} \sqsubseteq \mathrm{E}_1 \sqcap \mathrm{E}_2)^{\text{simp}} &= \{\mathrm{D} \sqsubseteq C_1 \sqcap C_2\} \cup (C_1 \sqsubseteq \mathrm{E}_1)^{\text{simp}} \cup (C_2 \sqsubseteq \mathrm{E}_2)^{\text{simp}} \\
(\mathrm{D} \sqsubseteq \mathrm{E}_1 \sqcup \mathrm{E}_2)^{\text{simp}} &= \{\mathrm{D} \sqsubseteq C_1 \sqcup C_2\} \cup (C_1 \sqsubseteq \mathrm{E}_1)^{\text{simp}} \cup (C_2 \sqsubseteq \mathrm{E}_2)^{\text{simp}} \\
(\mathrm{D} \sqsubseteq \forall f.\mathrm{E}_1)^{\text{simp}} &= \{\mathrm{D} \sqsubseteq \forall f.C_1\} \cup (C_1 \sqsubseteq \mathrm{E}_1)^{\text{simp}}
\end{aligned}$$

for $\mathrm{D} \sqsubseteq \mathrm{D}'$ a simple inclusion dependency and C_1 and C_2 fresh primitive concepts. We define $\mathcal{T}^{\text{simp}} = \bigcup_{\mathrm{D} \sqsubseteq \mathrm{E} \in \mathcal{T}} (\mathrm{D} \sqsubseteq \mathrm{E})^{\text{simp}}$.

Lemma 10

1. *Let $\mathcal{I} \models \mathcal{T}^{\text{simp}}$. Then $\mathcal{I} \models \mathcal{T}$;*
2. *Let $\mathcal{I} \models \mathcal{T}$. Then there is $\mathcal{I}'$ over the same domain such that $\mathcal{I}$ and $\mathcal{I}'$ agree on the interpretation of all symbols in $\mathcal{T}$ and $\mathcal{I}' \models \mathcal{T}^{\text{simp}}$.*

Proof Follows by a straightforward induction on the definition of $(\cdot)^{\text{simp}}$. □

Thus, in terminologies, the interaction of positive concept constructors with PFDs poses little difficulty and we can use existing decision procedures for the implication problem.

Theorem 11 *Let $\mathcal{T}$ be a terminology conforming to limited $\mathcal{DLFD}$ and $\mathcal{C}$ a simple inclusion dependency. Then $\mathcal{T} \models \mathcal{C}$ is decidable and complete for EXPTIME.*

Proof The theorem is a consequence of Lemma 10 and of reductions presented in Toman and Weddell [17, 19]. □

4.2 Transformation of Posed Questions

Now assuming, w.l.o.g., that a given terminology is simple, we exhibit a reduction of a logical implication problem with a posed question expressed in limited $\mathcal{DLFD}$. Unfortunately, allowing other than simple inclusion dependencies as posed questions leads to additional complications, as the following examples illustrate.

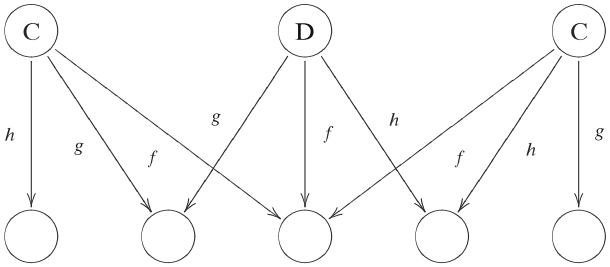

Fig. 4 Counterexample for Example 12

Example 12 A counterexample to $\mathrm{D} \sqsubseteq (\mathrm{C} : f, g \to h) \sqcup (\mathrm{C} : f, h \to g)$ is depicted in Fig. 4. Note that any such counterexample must also falsify $\mathrm{C} \sqsubseteq \mathrm{C} : f \to Id$ because distinct C objects that agree on f will be required. Thus,

$$\{\mathrm{C} \sqsubseteq \mathrm{C} : f \to Id\} \models \mathrm{D} \sqsubseteq (\mathrm{C} : f, g \to h) \sqcup (\mathrm{C} : f, h \to g).$$

Example 13 A counterexample to $\mathrm{D} \sqsubseteq (\mathrm{C} : f \to g) \sqcup \forall f.(\mathrm{C} : f \to g)$ is shown in Fig. 5. Observe with this case that distinct C objects must occur at *different* levels when compared to a D-rooted tree.

The examples suggest a need for multiple *root objects* in counterexample interpretations, with the roots themselves occurring at different levels. The overall strategy is to therefore reduce a logical implication problem to a negation of a consistency problem in an alternative formulation in which objects in a satisfying counterexample *denote up to ℓ possible copies in a counterexample interpretation for the original problem*, where ℓ is the number of occurrences of PFDs in the posed question.

To encode this one-to-many mapping of objects, we require a general way to have ℓ copies of concepts occurring in a given membership problem. We therefore write D^i to denote the concept description D in which all primitive concepts C are replaced by C^i. For a simple terminology $\mathcal{T}$ we then define

$$\mathcal{T}^i = \{\mathsf{Nd}^i \sqcap \mathrm{D}^i \sqsubseteq \mathrm{E}^i \mid \mathrm{D} \sqsubseteq \mathrm{E} \in \mathcal{T},\ E \text{ a non PFD}\},\ \text{and}$$

$$\begin{aligned}\mathcal{T}^{i,j} = \{&\mathsf{Nd}^i \sqcap \mathsf{Nd}^j \sqcap \mathrm{D}^i \sqcap \mathrm{E}^j \sqcap (\sqcap_{1 \le n \le k} \forall\, \mathsf{Pf}_n.\mathsf{Eq}^{i,j}) \sqsubseteq \forall\, \mathsf{Pf}.\mathsf{Eq}^{i,j},\\ &\mathsf{Nd}^i \sqcap \mathsf{Nd}^j \sqcap \mathrm{D}^j \sqcap \mathrm{E}^i \sqcap (\sqcap_{1 \le n \le k} \forall\, \mathsf{Pf}_n.\mathsf{Eq}^{i,j}) \sqsubseteq \forall\, \mathsf{Pf}.\mathsf{Eq}^{i,j}\\ &\mid \mathrm{D} \sqsubseteq \mathrm{E} : \mathsf{Pf}_1, \ldots, \mathsf{Pf}_k \to \mathsf{Pf} \in \mathcal{T}\ \}.\end{aligned}$$

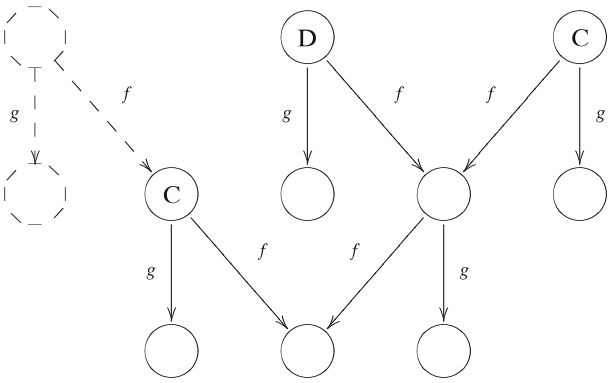

Fig. 5 Counterexample for Example 13

For a concept description E we define

$$\mathrm{Not}(\mathrm{E}) = \begin{cases} \neg \mathrm{D}^0 & \text{if E } (= \mathrm{D}) \text{ is free of PFDs,} \\ \mathrm{Not}(\mathrm{E}_1) \sqcap \mathrm{Not}(\mathrm{E}_2) & \text{if } \mathrm{E} = \mathrm{E}_1 \sqcup \mathrm{E}_2, \\ \mathrm{Not}(\mathrm{E}_1) \sqcup \mathrm{Not}(\mathrm{E}_2) & \text{if } \mathrm{E} = \mathrm{E}_1 \sqcap \mathrm{E}_2, \\ \forall f.\mathrm{Not}(\mathrm{E}_1) & \text{if } \mathrm{E} = \forall f.\mathrm{E}_1, \\ \mathsf{Nd}^i \sqcap \mathrm{D}^i \sqcap (\sqcap_{1 \le i \le k} \forall \mathsf{Pf}_i.\mathsf{Eq}^{0,i}) \sqcap \forall \mathsf{Pf}.\neg \mathsf{Eq}^{0,i} & \\ & \text{otherwise, when } \mathrm{E} = \mathrm{D} : \mathsf{Pf}_1, \ldots, \mathsf{Pf}_k \to \mathsf{Pf}. \end{cases}$$

In the last equation, i is the index of the PFD in the original posed question.

In the above, we have introduced primitive concepts $\mathsf{Eq}^{i,j}$, $0 \le i \ne j \le \ell$, to express that the ith and jth object copies coincide, and Nd^i, $0 \le i \le \ell$, to assert that the ith copy exists. The following auxiliary sets of inclusion dependencies are therefore defined to account for the axioms of equality and for the fact that attributes in $\mathcal{DLFD}$ denote total functions.

$$\begin{aligned} \mathcal{E}(l) \;=\; & \{\mathsf{Eq}^{i,j} \sqcap \mathsf{Eq}^{j,k} \sqsubseteq \mathsf{Eq}^{i,k} \mid 0 \le i < j < k \le \ell\} \\ & \cup\; \{\mathsf{Eq}^{i,j} \sqsubseteq \mathsf{Eq}^{j,i} \mid 0 \le i < j \le \ell\} \\ & \cup\; \{(\mathsf{Eq}^{i,j} \sqcap \mathrm{C}^i) \sqsubseteq \mathrm{C}^j \mid 0 \le i \ne j \le \ell \text{ and C a primitive concept}\} \\ & \cup\; \{\mathsf{Eq}^{i,j} \sqsubseteq \forall f.\mathsf{Eq}^{i,j} \mid 0 \le i \ne j \le \ell \text{ and } f \text{ a primitive feature}\} \\ \mathcal{N}(l) \;=\; & \{\mathsf{Nd}^i \sqsubseteq \forall f.\mathsf{Nd}^i \mid 0 \le i \le \ell \text{ and } f \text{ a primitive feature}\} \end{aligned}$$

Theorem 14 *Let $\mathcal{T}$ be a simple terminology and $\mathrm{D} \sqsubseteq \mathrm{E}$ an inclusion dependency containing ℓ occurrences of the PFD concept constructor. Then $\mathcal{T} \models \mathrm{D} \sqsubseteq \mathrm{E}$ if and only if*

$$\left(\bigcup_{0 \le i \le \ell} \mathcal{T}^i\right) \cup \left(\bigcup_{0 \le i < j \le \ell} \mathcal{T}^{i,j}\right) \cup \mathcal{E}(\ell) \cup \mathcal{N}(\ell) \models (\mathsf{Nd}^0 \sqcap \mathrm{D}^0 \sqcap \mathrm{Not}(\mathrm{E})) \sqsubseteq \bot.$$

Proof (sketch) Given an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \not\models \mathrm{D} \sqsubseteq \mathrm{E}$ we construct an interpretation $\mathcal{J}$ as follows. First, in the construction, we use a many-to-one map $\delta : \Delta \to \Delta^{\mathcal{J}}$ to associate objects in $\mathcal{I}$ with those in $\mathcal{J}$. The range of δ serves as the domain of the interpretation $\mathcal{J}$. For the counterexample object $o \in (\mathrm{D} \sqcap \neg \mathrm{E})^{\mathcal{I}}$, we set $\delta o \in (\mathsf{Nd}^0)^{\mathcal{J}}$. Then, for all $o \in \Delta$ and $0 \le i \ne j \le \ell$ we define the map δ and the interpretation $\mathcal{I}$ as follows:

- $\delta o \in (\mathsf{Nd}^i)^{\mathcal{J}} \wedge (f)^{\mathcal{I}}(o) = o' \Rightarrow \delta o' \in (\mathsf{Nd}^i)^{\mathcal{J}} \wedge (f)^{\mathcal{J}}(\delta o) = \delta o'$,
- $\delta o \in (\mathsf{Nd}^i)^{\mathcal{J}} \wedge o \in (D)^{\mathcal{I}} \Rightarrow \delta o \in (D)^{\mathcal{J}}$ for D a PFD-free concept,
- $\delta o = \delta o' \wedge \delta o \in (\mathsf{Nd}^i)^{\mathcal{J}} \wedge \delta o' \in (\mathsf{Nd}^j)^{\mathcal{J}} \wedge (\mathsf{Pf})^{\mathcal{I}}(o) = (\mathsf{Pf})^{\mathcal{I}}(o') \Rightarrow \;\; \delta o \in (\mathsf{Eq}^{i,j})^{\mathcal{J}}$, and
- $\delta o \in (\mathsf{Nd}^i)^{\mathcal{J}} \wedge o \in (\neg \mathrm{D} : \mathsf{Pf}_1, \ldots, \mathsf{Pf}_k \to \mathsf{Pf})^{\mathcal{I}}$ where $\mathrm{D} : \mathsf{Pf}_1, \ldots, \mathsf{Pf}_k \to \mathsf{Pf}$ is the i-th PFD constructor in E. Thus, there must be $o' \in \Delta$ such that $o' \in (\mathrm{D})^{\mathcal{I}}$ and the pair o and o' agree on all Pf_i but disagree on Pf; we set $\delta o = \delta o'$ and $\delta o' \in (\mathsf{Nd}^i \sqcap \mathrm{D}^i \sqcap (\sqcap_{1 \le i \le k} \forall \mathsf{Pf}_i.\mathsf{Eq}^{0,i}) \sqcap \forall \mathsf{Pf}.\neg \mathsf{Eq}^{0,i})^{\mathcal{J}}$.

Note that the syntactic restrictions imposed on the uses of PFD constructors imply that a negation of a PFD can be enforced only in the counterexample of the description E. Spurious occurrences of negated PFDs in the interpretation $\mathcal{I}$ are therefore ignored, since the interpretation itself satisfies all PFDs in $\mathcal{T}$.

One can easily verify that $\delta o \in (\mathsf{Nd}^0 \sqcap \mathrm{D}^0 \sqcap \mathrm{Not(E)})^{\mathcal{J}}$ for $o \in (\mathrm{D} \sqcap \neg \mathrm{E})^{\mathcal{I}}$. By inspecting all inclusion dependencies in $\mathcal{T}$ we have $\mathcal{J} \models \mathcal{T}^i$ as $\mathcal{I} \models \mathcal{T}$. Furthermore, the construction of $\mathcal{J}$ enforces $\mathcal{J} \models \mathcal{E}(\ell) \cup \mathcal{N}(\ell)$.

Conversely, given an interpretation $\mathcal{J}$ of $(\mathsf{Nd}^0 \sqcap \mathrm{D}^0 \sqcap \mathrm{Not(E)})$ that satisfies all inclusion dependencies in

$$\left(\bigcup_{0 \leq i \leq \ell} \mathcal{T}^i\right) \cup \left(\bigcup_{0 \leq i < j \leq \ell} \mathcal{T}^{i,j}\right) \cup \mathcal{E}(\ell) \cup \mathcal{N}(\ell),$$

we construct an interpretation $\mathcal{I}$ of $\mathcal{T}$ that falsifies $\mathrm{D} \sqsubseteq \mathrm{E}$ as follows:

- $\Delta^{\mathcal{I}} = \{(o, i) : o \in (\mathsf{Nd}^i)^{\mathcal{J}}, 0 \leq i \leq \ell \text{ and } o \notin (\mathsf{Eq}^{j,i})^{\mathcal{J}} \text{ for any } 0 \leq j < i\}$,
- $(f)^{\mathcal{I}}((o, i)) = (o', j)$ whenever $(f)^{\mathcal{J}}(o) = o'$ where j is the smallest integer such that $o \in (\mathsf{Eq}^{j,i})^{\mathcal{J}}$ if such value exists and i otherwise; and
- $(o, i) \in (\mathrm{D})^{\mathcal{I}}$ whenever $(o, i) \in \Delta^{\mathcal{J}}$ and $o \in (\mathrm{D}^i)^{\mathcal{J}}$.

One can easily verify that $(o, 0)$ falsifies $\mathrm{D} \sqsubseteq \mathrm{E}$ whenever o belongs to $(\mathsf{Nd}^0 \sqcap \mathrm{D}^0 \sqcap \mathrm{Not(E)})$, and such an object must exist by our assumptions. Also, $\mathcal{I} \models \mathcal{T}$, since, by case analysis otherwise, there is a contradiction with $\mathcal{J} \models \left(\bigcup_{0 \leq i \leq \ell} \mathcal{T}^i\right) \cup \left(\bigcup_{0 \leq i < j \leq \ell} \mathcal{T}^{i,j}\right) \cup \mathcal{E}(\ell) \cup \mathcal{N}(\ell)$.

Corollary 15 *The implication problem for limited $\mathcal{DLFD}$ is decidable and EXPTIME-complete.*

Proof Follows immediately from Theorems 11 and 14 above. □

5 On PFDs, Nominals, and ABoxes

In this section, we explore the possibility of relaxing the nonemptiness condition for left-hand sides of PFDs in limited $\mathcal{DLFD}$. Doing so is highly desirable because, as we have hinted in the introductory comments, this would effectively endow limited $\mathcal{DLFD}$ with a capability for *nominals*. To see this, consider that our introductory *single site for client information* example can be elaborated as follows.

$$\mathrm{Site3} \sqsubseteq \mathrm{Site3} :\rightarrow \mathit{Id} \quad \top \sqsubseteq \forall \mathit{Site3Ref}.\mathrm{Site3} \quad \mathrm{Client} \sqsubseteq \forall \mathit{LocationOfData}.\mathrm{Site3}$$

The first pair of inclusion dependencies define an individual called Site3 in two steps: (1) establish that at most one such individual exists, and (2) establish that at least one exists if anything exists. The final inclusion dependency then asserts that the location of data for any given client is this individual, thus accomplishing the objectives.

We show in the remainder of this section that allowing PFDs with empty left-hand sides—although desirable—leads to undecidability of the logical implication problem for limited $\mathcal{DLFD}$. To do so, we digress to consider a weaker alternative in which the problem is considered in the context of an ABox, showing for this case to begin with that the problem already becomes undecidable by presenting a reduction of the unrestricted tiling problem. However, it is possible with enough restrictions on the use of other concept constructors to reobtain decidability [9].

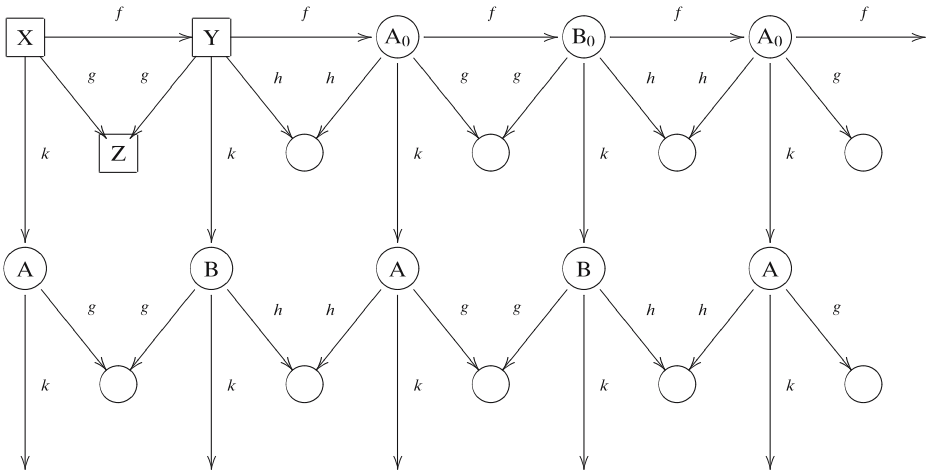

Fig. 6 Defining a grid using an ABox

An ABox consists of a finite collection of assertions $\mathcal{A}$ about individuals a_1, a_2, and so forth that denote elements of the domain. An assertion establishes concept membership for individuals with the form "$\mathrm{D}(a_i)$", or attribute values for individuals with the form "$f(a_i, a_j)$". The reduction of a given tiling problem $U = (T, H, V)$ to a logical implication problem for limited $\mathcal{DLFD}$ in the context of an ABox constructs a terminology and ABox pair, denoted $\langle \mathcal{T}_U, \mathcal{A}_U \rangle$, by first establishing an integer grid in steps.

1. Define a triangle *seed* pattern by including in $\mathcal{A}_U$ the following assertions.

$$\mathrm{X}(a_1) \;\; \mathrm{Y}(a_2) \;\; \mathrm{Z}(a_3) \;\; f(a_1, a_2) \;\; g(a_1, a_3) \;\; g(a_2, a_3)$$

2. Use the triangle seed pattern to create a possibly infinite *horizontal sequence* of objects that are instances of alternating concepts A_0 and B_0. The results of this step are illustrated along the top of Fig. 6.

$$\mathrm{X} \sqsubseteq \mathrm{A}_0 \qquad \mathrm{Y} \sqsubseteq \mathrm{B}_0$$

$$\mathrm{A}_0 \sqsubseteq \mathrm{A} \sqcap (\forall f.\mathrm{B}_0) \sqcap (\mathrm{B}_0 : g \rightarrow \; f \ldots h) \qquad \mathrm{B}_0 \sqsubseteq \mathrm{B} \sqcap (\forall f.\mathrm{A}_0) \sqcap (\mathrm{A}_0 : h \rightarrow \; f \ldots g)$$

3. Extend this sequence in the vertical direction to form the integer grid. The results of this step are also illustrated in Fig. 6.

$$\mathrm{A} \sqsubseteq (\forall k.\mathrm{A}) \sqcap (\mathrm{B} : g \rightarrow k \ldots g) \qquad \mathrm{B} \sqsubseteq (\forall k.\mathrm{B}) \sqcap (\mathrm{A} : h \rightarrow k \ldots h)$$

As before, the tiling problem U is modeled using primitive concepts T_i for each tile $t_i \in T$, asserting that $\mathrm{T}_i \sqcap \mathrm{T}_j \sqsubseteq \bot$ for all $i < j$. We place the tiles on the grid points with the inclusion dependency $(\mathrm{A} \sqcup \mathrm{B}) \sqsubseteq \bigsqcup_{t_i \in T} \mathrm{T}_i$. The adjacency rules for the instance U of the tiling problem are then captured as follows:

– for $(t_i, t_j) \notin H$:

$$\mathrm{A} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{B} \sqcap \mathrm{T}_j) : g \rightarrow Id \qquad \mathrm{B} \sqcap \mathrm{T}_i \sqsubseteq (\mathrm{A} \sqcap \mathrm{T}_j) : h \rightarrow Id$$

– for $(t_i, t_j) \notin V$:

$$(\mathrm{A} \sqcap \mathrm{T}_i) \sqcap \forall k.(\mathrm{A} \sqcap \mathrm{T}_j) \sqsubseteq \bot \qquad (\mathrm{B} \sqcap \mathrm{T}_i) \sqcap \forall k.(\mathrm{B} \sqcap \mathrm{T}_j) \sqsubseteq \bot$$

where T_i corresponds to a tile type $t_i \in T$. The above inclusion dependencies form a terminology $\mathcal{T}_U$ and ABox $\mathcal{A}_U$ associated with an unrestricted tiling problem U that immediately yields the following result.

Theorem 16 *An instance $U = (T, H, V)$ of the infinite tiling problem admits a solution if and only if*

$$\langle \mathcal{T}_U, \mathcal{A}_U \rangle \not\models \mathrm{X} \sqsubseteq \bot.$$

Corollary 17 *The logical implication problem for limited $\mathcal{DLFD}$ in the context of an ABox is undecidable.*

The main result in this section now follows because PFDs with empty left-hand sides can simulate the above triangle seed by instead adding the following to $\mathcal{T}_U$.

$$\mathrm{X} \sqcap \mathrm{Y} \sqsubseteq \bot \quad \mathrm{X} \sqsubseteq (\forall f.\mathrm{Y}) \sqcap (\forall g.\mathrm{Z}) \quad \mathrm{Y} \sqsubseteq \forall g.\mathrm{Z} \quad \mathrm{Z} \sqsubseteq \mathrm{Z} :\rightarrow Id$$

Corollary 18 *The logical implication problem for limited $\mathcal{DLFD}$ in which PFDs are permitted empty left-hand sides is undecidable.*

6 Conclusions

We have shown that allowing PFDs to occur in the scope of negation or on the left-hand sides of inclusion dependencies in $\mathcal{DLFD}$ leads to undecidability of its logical implication problem and therefore that a full integration of keys and functional dependencies in expressive DLs is not, in general, possible. Conversely, by virtue of reductions to simpler dialects, we have shown that the complexity of this problem remains unchanged for limited $\mathcal{DLFD}$ in which PFDs are restricted to occur within the scope of monotone concept constructors on right-hand sides of inclusion dependencies.

Limited $\mathcal{DLFD}$ can be extended in several ways without changing the complexity of its logical implication problem. For example, by using reductions introduced in Toman and Weddell [19], it is straightforward to add roles, quantified number restrictions on roles and even role inversion. (Feature inversion, however, is another matter since its addition to simple $\mathcal{DLFD}$ already leads to undecidability [18, 20].)

There is also a possibility of extending limited $\mathcal{DLFD}$ with *regular path functional dependencies* as defined in Toman and Weddell [17]. In this case, left- and right-hand sides of PFDs are specified as regular languages that can define infinite sets of path expressions. Such inclusion dependencies have applications in reasoning about equality in semistructured databases [17] and in capturing inductive data types in information integration, thus extending the work in [12].

Another direction of future research includes studying terminologies stratified with respect to the interactions of the PFD constructor and negation in an attempt to extend the applicability of the proposed approach.

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