Description Logics and Time

Jie Zhang

David R. Cheriton School of Computer Science
University of Waterloo

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Outline

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Knowledge Representation

- An important role in knowledge-based systems
  - Rule-based systems
    - Use rules as the knowledge representation for knowledge coded into the system
  - Case-based systems
    - Expertise is embodied in a library of past cases, rather than being encoded in classical rules
    - Each case contains a description of problem + a solution or the outcome
    - Reasoning process used by an expert to solve the problem is not recorded, but is implicit in the solution.

Knowledge Representation

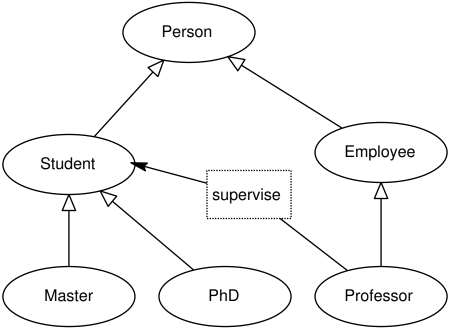
- Tasks of Knowledge Representation
  - Representing knowledge base of application domain, aka the World
  - Reasoning to infer implicitly represented knowledge from the knowledge that is explicitly contained in the knowledge base
- Old Approaches
  - Logic-based formalisms
  - Non-logic-based representation: Network-based structure

Logic-based Formalisms

- Based on first-order logic
  - Knowledge is represented by first-order logic predicate calculus, reasoning to verifying logical consequence
- Why not propositional logic
  - Cannot access the structure of atomic sentences
  - Example: jack is a master student
    - In propositional logic: considered as a whole
    - In first-order logic: Master (jack), very expressive
- Problems
  - Expressive power is too high for obtaining decidable and efficient inference problems
  - Inference power may be too low for expressing interesting, but still decidable theories

Non-logic-based representation

- Based on graphical interfaces
- Semantic networks and frames: Network-based Structure



Non-logic-based representation

- Advantages
  - Straight forward
  - Effective
  - More appealing
- Problems
  - Lack precise semantic characterization
    - Represented semantics are ambiguous
    - Arcs can represent different kinds of relations
    - Systems developed behave differently
  - Reasoning on it is inefficient

Description Logics

- Combination of both approaches (Hayes, 1979)
  - Giving semantics to network-based structures by using first-order logics
  - A fragment of first-order logic
- The Result: Description Logics
  - Concept: a description of a collection of individuals with common properties, i.e. Student
  - Roles: interrelationships between individuals, i.e. supervise
  - Individuals: constants of concepts, i.e. Jack is a constant of Master student

Different Versions

- $\mathcal{FL}^-$(Brachman and Levesque in 1984)
  - Concept conjunction, universal role quantification and limited existential quantification
- $\mathcal{AL}$ (Schmidt-SchauB and Smolka in 1991)
  - Minimal language of practical interest
  - Adds negation constructor
- $\mathcal{FL}$: disallows limited existential quantification
- $\mathcal{ALC}$: adds full negation and disjunction on $\mathcal{FL}$
- $\mathcal{ALCF}$: adds functions on $\mathcal{ALC}$, such as agreement and disagreement, aka features
- $\mathcal{AL\varepsilon N}$: full existential quantification, number restrictions

Syntax

$$C, D \rightarrow A | C \sqcap D | C \sqcup D | \neg C | \forall R.C | \exists R.C$$

- A is an atomic concept
- R is an atomic role
- C and D are concepts
- Examples
  - $Person \sqcap Student$: those persons who are students
  - $Person \sqcap \forall supervise.PhD$: those persons all of whom supervise PhD students
  - $Person \sqcap \exists supervise.Student$: those persons at least one of whom supervises students

Semantics

- $\cdot^{\mathcal{I}}$: an interpretation function maps
  - Concept to a subset of $\triangle^{\mathcal{I}}$
  - Role to a subset of $\triangle^{\mathcal{I}} \times \triangle^{\mathcal{I}}$
  - Every individual to an element of $\triangle^{\mathcal{I}}$

Syntax	Formal Semantics	FOL semantics
A :	$A^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}}$	$F_A(x)$
$C \sqcap D$:	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$F_C(x) \wedge F_D(x)$
$C \sqcup D$:	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$	$F_C(x) \vee F_D(x)$
$\neg C$:	$\triangle^{\mathcal{I}} \backslash C^{\mathcal{I}}$	$\neg F_C(x)$
$\forall R.C$:	$\{a \in \triangle^{\mathcal{I}} \mid \forall b.(a,b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$	$\forall z.F_R(x,z) \rightarrow F_c(z)$
$\exists R.C$:	$\{a \in \triangle^{\mathcal{I}} \mid \exists b.(a,b) \in R^{\mathcal{I}}\} \wedge b \in C^{\mathcal{I}}\}$	$\exists z.F_R(x,z) \wedge F_c(z)$

Reasoning Services

- Concept satisfiability
  - A concept C is satisfiable if there exists a model $\mathcal{I}$ such that $C^{\mathcal{I}} \neq \emptyset$
- Subsumption
  - A concept C is subsumed by a concept D if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$.
- Other: instance checking, retrieval, realization
- Expressivity verses Decidability
  - Trade-off (Firstly argued by Brachman and Levesque)
  - The more expressive, the harder the reasoning will be

Relation to DB and Possible Extensions

- DL for Database Management
  - Strongly related
    - Knowledge-based Systems contain both database systems and description logics knowledge representation systems
  - Used to address many issues in DBM
    - Check concept consistency to verify at design time whether an entity can have at least one instance
    - Query optimization and/or information integration
- Possible Extensions of DL
  - Adding representational features
  - Adding reasoning services
  - Temporal extensions: handling time

Dimensions of Temporal DL

- Differ on notation of time
  - Point-based notion of time
  - Interval-based notion of time
- Differ on ways of adding notion of time
  - Implicitly: temporal information only implicit in language
  - Explicitly: an explict notion of time is adopted
- In an explicit representation of time
  - External point of view
    - Same individual can have different snapshots in different moments describing various states of the individual at these times
  - Internal point of view
    - diffent states of an individual are seen as different individual components

Pointed-based Temporal DL

- $\mathcal{ALCT}$ (Schild): $\mathcal{ALC}$ + time
- Syntax

$$
\begin{array}{rrl}
C, D \rightarrow & C\mathcal{U}D| & (C \text{ until } D) \text{ (until)} \\
& C\mathcal{S}D| & (C \text{ since } D) \text{ (since)} \\
& \Diamond C| & \text{sometime in the future} \\
& \Box C| & \text{always in the future} \\
& \blacklozenge C| & \text{sometime in the past} \\
& \blacksquare C| & \text{always in the past}
\end{array}
$$

- Define connectives using Since and Until

$$\Diamond C \doteq \top \mathcal{U} C \qquad \blacklozenge C \doteq \top \mathcal{S} C$$

$$\Box C \doteq \neg \Diamond \neg C \qquad \blacksquare C \doteq \neg \blacklozenge \neg C$$

Pointed-based Temporal DL

- Semantics
  - $\mathcal{T} = (\mathcal{P}, <)$: $\mathcal{P}$ is a set of time points; $<$ is a strict linear order
  - $\mathcal{M} \doteq \langle \mathcal{T}, \mathcal{I} \rangle$: $\mathcal{I}$ is a function associating to each $t \in \mathcal{P}$ a standard non-temporal $\mathcal{ALC}$ interpretation, $\mathcal{I}(t) \doteq \langle \triangle^{\mathcal{I}}, \cdot^{\mathcal{I}(t)} \rangle$

$$(C\mathcal{U}D)^{\mathcal{I}(t)} = \{x \in \triangle^{\mathcal{I}} | \exists v.(v > t) \wedge D^{\mathcal{I}(v)}(x) \wedge \forall w.(t < w < v) \rightarrow C^{\mathcal{I}(w)}(x)\}$$

$$(C\mathcal{S}D)^{\mathcal{I}(t)} = \{x \in \triangle^{\mathcal{I}} | \exists v.(v > t) \wedge D^{\mathcal{I}(v)}(x) \wedge \forall w.(v < w < t) \rightarrow C^{\mathcal{I}(w)}(x)\}$$

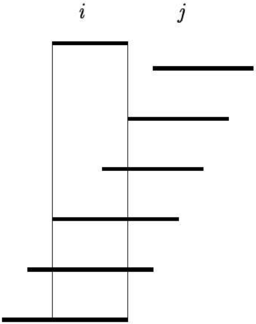
- Example

$$PhD \doteq Person \sqcap (Student\mathcal{U}\square Professor)$$

Interval-based Temporal DL

- Firstly proposed by Schmiedel
- Based on Allen's interval relationships

Relation	Abbr.	Inverse
before(i, j)	b	a
meets(i, j)	m	mi
overlaps(i, j)	o	oi
starts(i, j)	s	si
during(i, j)	d	di
finishes(i, j)	f	fi



Interval-based Temporal DL

- Semantics
  - An Interval of $\mathcal{T}$, $\mathcal{T}^*$: $[t_1, t_2] \doteq \{t \in \mathcal{P} | t_1 \leq t \leq t_2\}$
  - $\mathcal{M} \doteq \langle \mathcal{T}^*, \mathcal{I} \rangle$: $\mathcal{I}$ is a function associating to each $i = [t_1, t_2] \in \mathcal{T}^*$ a standard non-temporal $\mathcal{ALC}$ interpretation

$$(\langle \alpha \rangle C)^{\mathcal{I}(i)} = \{x \in \triangle^{\mathcal{I}} | \exists j.\alpha(j, i) \wedge C^{\mathcal{I}(j)}(x)\}$$

$$([\alpha]C)^{\mathcal{I}(i)} = \{x \in \triangle^{\mathcal{I}} | \forall j.\alpha(j, i) \rightarrow C^{\mathcal{I}(j)}(x)\}$$

- Example

$$PhD \doteq Person \sqcap (Student \sqcap < mi > Professor)$$

Decidability and Complexity

- Concept satisfiability
  - A concept is satisfiable if there exists $\mathcal{M}$ and t such that $\mathcal{M}, t \models \neg(C \doteq \bot)$, which means that there exists an interpretation $\mathcal{I}$ such that $C^{\mathcal{I}(t)} \neq \emptyset$ for some t
- Decidability
  - There exists a computational process that solves the problem in a finite number of steps
- No temporal roles for $\mathcal{ALCT}$
  - $\mathcal{ALCT}$ with global roles is undecidable in any unbounded linear order
- Complexity
  - concept satisfiability in $\mathcal{ALCT}$ without global roles is *PSPACE – complete*

Summary

- Introduction to the history of description logics
- Syntax and semantics of DL and other related issues: reasoning, relation to DB management, and possible extensions
- Syntax and semantics of temporal DL, including point-based and interval-based temporal DL, and the decidability and complexity issues

References

- Enrico Franconi's lecture notes: Foundations of first order logic.
- Alessandro Artale and Enrico Franconi. Introducing temporal description logics. 1999.
- Alessandro Artale and Enrico Franconi. A survey of temporal extensions of description logics. Annals of Mathematics and Artificial Intelligence. 2000.
- Alessandro Artale and Enrico Franconi. Temporal description logics. 2000.
- F. Baader and W. Nutt. Basic description logics. 2002.
- Jan Chomicki and David Toman. Temporal logic in information systems. 1998.
- Francesco M. Donini, Maurizio Lenzerini, Daniele Nardi, and Andrea Schaerf. Reasoning in description logics. 1996.
- J. Kelly. The Essence of Logic. Prentice Hall, 1997.
- H. J. Levesque and R. J. Brachman. Expressiveness and tractability in knowledge representation and reasoning. 1987.
- D. Nardi and R. J. Brachman. An introduction to description logics. 2002.

Thank You!