

Module 2: The Relational Model

Winter 2026

Cheriton School of Computer Science

CS 348: Intro to Database Management


Reading Assignments and References

To be read during the Week of January 12–16:

- ▶ Chapter 2 of course textbook.¹ (Material in Sections 2.3 and 2.6 will be covered in later modules.)
- ▶ Section 27.2 of Chapter 27 of course textbook, available online at db-book.com.

References

1. Abiteboul, Hull and Vianu, *Foundations of Databases*. A book available online at <http://webdam.inria.fr/Alice/>.
2. *Backus-Naur Form*, wiki page.

¹Silberschatz, Korth and Sudarshan, *Database Systems Concepts*, 7th edition 

Outline

Unit 1: **Signatures and the Relational Calculus**

Unit 2: Integrity Constraints

Unit 3: Safety and Finiteness

Unit 4: Summary

A Basic Syntax for Asking Questions and for Answers

To begin with, assume ...

- ▶ Set comprehension syntax for queries:

$$\{\langle \textit{answer} \rangle \mid \langle \textit{condition} \rangle\}.$$

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- ▶ Syntax for each $\langle \textit{answer} \rangle$ is a k -tuple of *variables*:

$$(x_1, \dots, x_k).$$

- ▶ Answers to a query:

all k -tuples (c_1, \dots, c_k) of constants denoting values for each variable x_i that satisfy $\langle \textit{condition} \rangle$.

Asking Questions about Natural Numbers

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Why? Because $(0, 5, 5)$, etc., appear in table PLUS!

Table PLUS

A1	A2	R
0	0	0
0	1	1
0	2	2
⋮	⋮	⋮
1	0	1
1	1	2
⋮	⋮	⋮
2	0	2
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What are all pairs of numbers that add to the same number they subtract to, where $x + y = x - y$?

Question: $\{(x, y) \mid \exists z. \text{PLUS}(x, y, z) \wedge \text{PLUS}(z, y, x)\}$

Answers: $\{(0, 0), (1, 0), \dots\}$

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What is the neutral element of addition?

Question: $\{(x) \mid \text{PLUS}(x, x, x)\}$

Answers: $\{(0)\}$

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Asking Questions about Employees

Who are all the employees and their departments who work for Bob?

Question: $\{(x, y) \mid \text{EMP}(x, y, \text{Bob})\}$

Table EMP

name	dept	boss
<i>Sue</i>	CS	<i>Bob</i>
<i>Bob</i>	CO	<i>Bob</i>
<i>Fred</i>	PM	<i>Mark</i>
<i>John</i>	PM	<i>Mark</i>
<i>Jim</i>	CS	<i>Fred</i>
<i>Eve</i>	CS	<i>Fred</i>
<i>Sue</i>	PM	<i>Sue</i>

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Q: $\{(x_1, x_2) \mid \exists y_1, y_2, z. \text{EMP}(x_1, y_1, z) \wedge \text{EMP}(x_2, y_2, z)\}$

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Who are the employees who are their own bosses?

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Based on *first order predicate logic* (FOL) and *Tarskian semantics*.

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Recall example RM database using a common visualization:

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WROTE

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1	4
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PUBLICATION

pubid	title
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3	Trans. on Databases
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4	Query Languages

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- ▶ declarative DML based on *well-formed formulas* in FOL, and
- ▶ *integrity constraints* also via well-formed formulas.

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Universe ▶ a set of **values** **D** (*domain*) with *equality* (=)

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Notation

Signature: $\rho = (R_1/k_1, \dots, R_n/k_n)$

Instance: $\mathbf{DB} = (\mathbf{D}, =, \mathbf{R}_1, \dots, \mathbf{R}_n)$

Examples of Relational Databases

- ▶ The integers, with addition *and multiplication*:

(signature) $\rho = (\text{PLUS}/3, \text{TIMES}/3)$

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(data) **DB** = (STR , =, **EMP**)

- ▶ The simple bibliography database:

(signature) $\rho = (\text{AUTHOR}/2, \text{WROTE}/2, \text{PUBLICATION}/2)$

(data) **DB** = ($\text{STR} \uplus \mathbb{Z}$, =, **AUTHOR**, **WROTE**, **PUBLICATION**)

Bibliography Relational Database, Version 2

(signature) $\rho = ($

```
AUTHOR (aid, name),  
WROTE (author, publication),  
PUBLICATION (pubid, title),  
BOOK (pubid, publisher, year),  
JOURNAL-OR-PROCEEDINGS (pubid),  
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Arity is indicated by a sequence of *identifiers*, called **attributes**:

- ▶ Help with understanding semantics; and
- ▶ Used in some DMLs, such as some *relational algebras* and SQL.

Bibliography Relational Database, Version 2 (cont'd)

(data) **DB** = (STR \cup Z, =,

AUTHOR = { (1, Sue), (2, John) },

WROTE = { (1, 1), (1, 4), (1, 2), (2, 2) },

PUBLICATION = { (1, Mathematical Logic),
(3, Trans. on Databases),
(2, Principles of DB Systems),
(4, Query Languages) },

BOOK = { (1, AMS, 1990) },

JOURNAL-OR-PROCEEDINGS = { (2), (3) },

JOURNAL = { (3, 35, 1, 1990) },

PROCEEDINGS = { (2, 1995) },

ARTICLE = { (4, 2, 30, 41) }

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- ▶ John wrote “Principles of DB Systems” since: $(2, 2) \in \mathbf{WROTE};$
- ▶ John has **NOT** written “Trans. on Databases” since: $(2, 3) \notin \mathbf{WROTE};$
- ▶ etc.

Query Conditions

Idea

Use **variables** and **valuations** to generalize conditions.

Example: $\text{AUTHOR}(x, y)$ will be true of any *valuation* $\{x \mapsto v_1, y \mapsto v_2, \dots\}$ exactly when the 2-tuple of values (v_1, v_2) occurs in **AUTHOR**.

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Example: $\text{AUTHOR}(x, y)$ will be true of any *valuation* $\{x \mapsto v_1, y \mapsto v_2, \dots\}$ exactly when the 2-tuple of values (v_1, v_2) occurs in **AUTHOR**.

Valuation

A valuation is a function θ that maps *variable names* to values in the universe:

$$\theta : \{x_1, x_2, \dots\} \rightarrow \mathbf{D}.$$

Query Conditions

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To denote a modification to θ in which variable x is instead mapped to value v , one writes:

$$\theta[x \mapsto v].$$

Query Conditions (cont')

Idea

Allow more complex conditions to be built from simpler conditions with ...

Logical connectives:

Conjunction (and): $\text{AUTHOR}(x, y) \wedge \text{WROTE}(x, z)$

Disjunction (or): $\text{AUTHOR}(x, y) \vee \text{PUBLICATION}(x, y)$

Negation (not): $\neg \text{AUTHOR}(x, y)$

Quantifiers:

Existential (there is...): $\exists x. \text{author}(x, y)$

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Examples:

▶ $\exists z. \text{PLUS}(x, y, z) \wedge \text{PLUS}(z, y, x)$

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Examples:

- ▶ $\exists z. \text{PLUS}(x, y, z) \wedge \text{PLUS}(z, y, x)$, or
- ▶ $\exists y_1, y_2, z. \text{EMP}(x_1, y_1, z) \wedge \text{EMP}(x_2, y_2, z)$.

Query Conditions (cont')

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Summarizing, allow conditions to be *well-formed formulas* (wffs) in the language of FOL.

Relational Calculus

Conditions

Given a database signature $\rho = (R_1/k_1, \dots, R_n/k_n)$, a set of variable names $\{x_1, x_2, \dots\}$ and a set of constants $\{c_1, c_2, \dots\}$, *conditions* are *formulas* defined by the grammar:

$$\varphi ::= \underbrace{R_i(x_{i,1}, \dots, x_{i,k_i}) \mid x_i = x_j \mid x_i = c_j \mid \varphi_1 \wedge \varphi_2 \mid \exists x_i. \varphi_1 \mid \varphi_1 \vee \varphi_2 \mid \neg \varphi_1}_{\text{conjunctive formulas}}$$
$$\underbrace{\hspace{10em}}_{\text{positive formulas}}$$
$$\underbrace{\hspace{15em}}_{\text{first-order formulas}}$$

A condition is a *sentence* when it has no **free variables**.

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positive formulas

first-order formulas

A condition is a *sentence* when it has no **free variables**.

The meta-language used to define the grammar is Backus-Naur form. (See wiki for a good overview.)

Relational Calculus (cont')

Free Variables

The *free variables* of a formula φ , written $Fv(\varphi)$, are defined as follows:

$$Fv(R(x_{i_1}, \dots, x_{i_k})) = \{x_{i_1}, \dots, x_{i_k}\};$$

$$Fv(x_i = x_j) = \{x_i, x_j\};$$

$$Fv(x_i = c_j) = \{x_i\};$$

$$Fv(\varphi \wedge \psi) = Fv(\varphi) \cup Fv(\psi);$$

$$Fv(\exists x_i. \varphi) = Fv(\varphi) - \{x_i\};$$

$$Fv(\varphi \vee \psi) = Fv(\varphi) \cup Fv(\psi); \text{ and}$$

$$Fv(\neg \varphi) = Fv(\varphi).$$

Semantics for Conditions

When a Condition is True (Tarski)

The *truth* of a formula φ over a signature $\rho = (R_1/k_1, \dots, R_n/k_n)$ is defined with respect to

1. a **database instance** $\mathbf{DB} = (\mathbf{D}, =, \mathbf{R}_1, \dots, \mathbf{R}_n)$, and
2. a **valuation** $\theta : \{x_1, x_2, \dots\} \rightarrow \mathbf{D}$

as follows:

$\mathbf{DB}, \theta \models R_i(x_{i,1}, \dots, x_{i,k_i})$	if $(\theta(x_{i,1}), \dots, \theta(x_{i,k_i})) \in \mathbf{R}_i$;
$\mathbf{DB}, \theta \models x_i = x_j$	if $\theta(x_i) = \theta(x_j)$;
$\mathbf{DB}, \theta \models x_i = c_j$	if $\theta(x_i) = c_j$;
$\mathbf{DB}, \theta \models \varphi \wedge \psi$	if $\mathbf{DB}, \theta \models \varphi$ and $\mathbf{DB}, \theta \models \psi$;
$\mathbf{DB}, \theta \models \exists x_i. \varphi$	if $\mathbf{DB}, \theta[x_i \mapsto v] \models \varphi$, for some $v \in \mathbf{D}$;
$\mathbf{DB}, \theta \models \varphi \vee \psi$	if $\mathbf{DB}, \theta \models \varphi$ or $\mathbf{DB}, \theta \models \psi$; and
$\mathbf{DB}, \theta \models \neg \varphi$	if $\mathbf{DB}, \theta \not\models \varphi$.

Equivalences and Syntactic Sugar

Boolean Equivalences

- ▶ $\neg(\neg\varphi_1) \equiv \varphi_1$
- ▶ $\varphi_1 \vee \varphi_2 \equiv \neg(\neg\varphi_1 \wedge \neg\varphi_2)$
- ▶ $\varphi_1 \rightarrow \varphi_2 \equiv \neg\varphi_1 \vee \varphi_2$
- ▶ $\varphi_1 \leftrightarrow \varphi_2 \equiv (\varphi_1 \rightarrow \varphi_2) \wedge (\varphi_2 \rightarrow \varphi_1)$
- ▶ ...

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First-order Equivalences

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- ▶ ...

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Additional Syntactic Sugar

- ▶ $R(\dots, c, \dots) \equiv \exists x.(R(\dots, x, \dots) \wedge x = c)$, where x is *fresh*
- ▶ $\exists x_1, \dots, x_n.\varphi \equiv \exists x_1.\dots.\exists x_n.\varphi$
- ▶ $R(\dots, -, \dots) \equiv \exists x.R(\dots, x, \dots)$, where x is *fresh*

Relational Calculus (cont'd)

Relational Calculus (RC) Query

A *query* in the relational calculus is a set comprehension of the form

$$\{(x_1, \dots, x_k) \mid \varphi\},$$

where $\{x_1, \dots, x_k\} = \text{Fv}(\varphi)$ (are the free variables of φ).

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Also:

- ▶ a **conjunctive query** is where φ is a conjunctive formula, and
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Query Answers

The *answers* to a query $\{(x_1, \dots, x_k) \mid \varphi\}$ over **DB** is the **relation**

$$\{(\theta(x_1), \dots, \theta(x_k)) \mid \mathbf{DB}, \theta \models \varphi\}.$$

Relational Calculus (cont'd)

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Answers to queries: valuations applied to tuples of variables that make the formula true with respect to a database.

Example Justification of an Answer to an RC Query

Who are pairs of employees working for the same boss?

Q: $\{(x_1, x_2) \mid \exists y_1, y_2, z. \text{EMP}(x_1, y_1, z) \wedge \text{EMP}(x_2, y_2, z)\}$

A: $\{(Jim, Eve), \dots\}$

Table EMP

name	dept	boss
<i>Sue</i>	CS	<i>Bob</i>
<i>Bob</i>	CO	<i>Bob</i>
<i>Fred</i>	PM	<i>Mark</i>
<i>John</i>	PM	<i>Mark</i>
<i>Jim</i>	CS	<i>Fred</i>
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Because:

1. **DB**, $\{x_1 \mapsto Jim, y_1 \mapsto CS, z \mapsto Fred, \dots\} \models \text{EMP}(x_1, y_1, z)$

where $\rho = (\text{EMP}/3)$, and **DB** = (STR, =, EMP).

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where $\rho = (\text{EMP}/3)$, and **DB** = (**STR**, =, **EMP**).

† Check that $\{x_1, x_2\} = \text{Fv}(\exists y_1, y_2, z. \text{EMP}(x_1, y_1, z) \wedge \text{EMP}(x_2, y_2, z))$.

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where $\rho = (\text{EMP}/3)$, and **DB** = (**STR**, =, **EMP**).

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More Examples of RC Queries

Over signature $\rho = (\text{EMP}(\text{name}, \text{dept}, \text{boss}))$:

1. *Who are the bosses that manage at least two employees?*

$$\{(b) \mid \exists e_1, e_2. (\exists d_1. \text{EMP}(e_1, d_1, b)) \wedge (\exists d_2. \text{EMP}(e_2, d_2, b)) \wedge \neg(e_1 = e_2)\}$$

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(or more simply, with the aid of some syntactic sugar)

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Choose variable names suggestive of what values or (indirectly) entities they refer to, e.g.:

- ▶ “ e_1 ” refers indirectly to an employee, and
- ▶ “ b ” refers indirectly to a boss.

Exercises

1. Over the PLUS-TIMES relational database, with
signature $\rho = (\text{PLUS}/3, \text{TIMES}/3)$, and
instance $\text{DB} = (\mathbb{N}, =, \text{PLUS}, \text{TIMES})$:[†]
 - 1.1 *What are all composite numbers?*
 - 1.2 *What are all prime numbers?*
2. Over the bibliography relational database, 2nd version:
 - 2.1 *What are all publication titles?*
 - 2.2 *What are the publication titles that are journals or proceedings?*
 - 2.3 *What are the titles of all books?*
 - 2.4 *What are the publications without authors?*
 - 2.5 *What are all the ordered pairs of coauthor names?*
 - 2.6 *What are all publication titles written by a single author?*

[†] Much harder over the integers \mathbb{Z} .

Outline

Unit 1: Signatures and the Relational Calculus

Unit 2: **Integrity Constraints**

Unit 3: Safety and Finiteness

Unit 4: Summary

Asking Questions about Natural Numbers (revisited)

What is the neutral element of addition?

Question: $\{(x) \mid \text{PLUS}(x, x, x)\}$

Answers: $\{(0)\}$

Table PLUS

A1	A2	R
0	0	0
0	1	1
0	2	2
⋮	⋮	⋮
1	0	1
1	1	2
⋮	⋮	⋮
2	0	2
2	1	3
⋮	⋮	⋮
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$\{(x) \mid \forall y. \text{PLUS}(x, y, y) \wedge \text{PLUS}(y, x, y)\}$? (*)

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Observation

(*) is the same as

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because PLUS is **commutative**!

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because PLUS is **commutative**! And (**) is the same as

$$\{(x) \mid \text{PLUS}(x, x, x)\}$$

because PLUS is **monotone**!

Table PLUS

A1	A2	R
0	0	0
0	1	1
0	2	2
⋮	⋮	⋮
1	0	1
1	1	2
⋮	⋮	⋮
2	0	2
2	1	3
⋮	⋮	⋮

Asking Questions about Natural Numbers (revisited)

What is the neutral element of addition?

Question: $\{(x) \mid \text{PLUS}(x, x, x)\}$

Answers: $\{(0)\}$

But shouldn't the query really be

$$\{(x) \mid \forall y. \text{PLUS}(x, y, y) \wedge \text{PLUS}(y, x, y)\} \quad (*)$$

Observation

(*) is the same as

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because PLUS is **commutative**! And (**) is the same as

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PLUS should satisfy **integrity constraints** that are the *laws of arithmetic* for natural numbers.

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Integrity Constraints for Addition

Sentences that should always be *true* for any extension of PLUS over the domain of natural numbers:

- ▶ *Addition is commutative:*
- ▶ *PLUS is a relational representation of a binary function:*
- ▶ *Addition is a total function:*
- ▶ *Addition is monotone in both arguments (harder), etc., etc.*

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Sentences that should always be *true* for any extension of table

EMP (name, dept, boss) :

- ▶ *Every boss is an employee:*
- ▶ *Every boss manages a unique department:*
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A relational *signature* captures only the structure of relations.

Valid database instances satisfy additional *integrity constraints* in the form of sentences over the signature.

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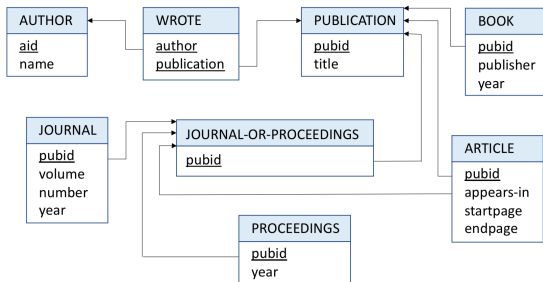
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- ▶ etc.

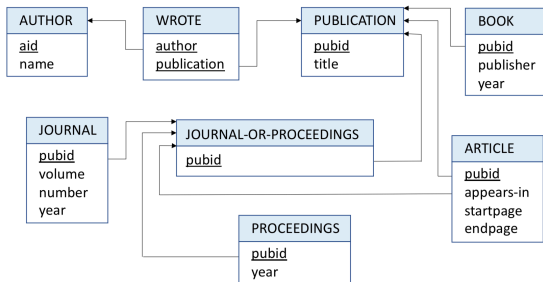
Bibliography Integrity Constraints



Typing Constraints / Domain Constraints

- ▶ *Author id's are integers.*
- ▶ *Author names are strings.*
- ▶ *Publication id's are integers.*
- ▶ *Publication titles are strings.*
- ▶ *etc.*

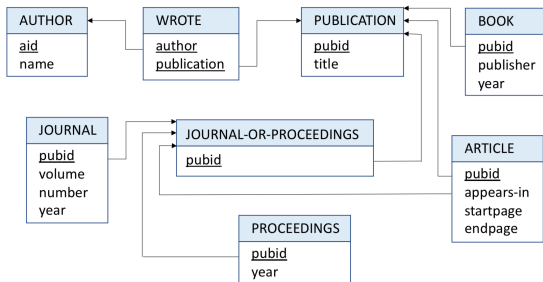
Bibliography Integrity Constraints (cont'd)



Uniqueness of Values / Identification (keys)

- ▶ *Author id's are unique and determine author names.*
- ▶ *Publication id's are unique as well.*
- ▶ *Articles can be identified by their publication id.*
- ▶ *Articles can also be identified by the publication id of the collection they have appeared in and their starting page number.*

Bibliography Integrity Constraints (cont'd)



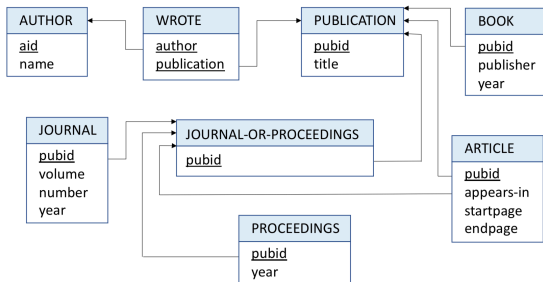
Referential Integrity / Foreign Keys

- ▶ *Books, journals, proceedings and articles are publications.*
- ▶ *The components of a WROTE tuple must be an author and a publication.*

Disjointness

- ▶ *Books are different from journals.*
- ▶ *Books are also different from proceedings.*

Bibliography Integrity Constraints (cont'd)



Coverage

- ▶ *Every publication is either a book, a journal, a proceedings, or an article.*
- ▶ *Every article appears in a journal or in a proceedings.*

Views and Integrity Constraints

The extension of a table can be determined by an integrity constraint.

The extension of table JOURNAL-OR-PROCEEDINGS is the union of the publication id's occurring in table JOURNAL and in table PROCEEDINGS.

$$\forall p. \text{JOURNAL-OR-PROCEEDINGS}(p) \leftrightarrow (\text{JOURNAL}(p) \vee \text{PROCEEDINGS}(p))$$

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View

Given a signature ρ , a table R occurring in ρ is a *view* when the **relational database schema** contains exactly one integrity constraint of the form:

$$\forall x_1, \dots, x_k. R(x_1, \dots, x_k) \leftrightarrow \varphi,$$

where $\{x_1, \dots, x_k\} = \text{Fv}(\varphi)$.

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No table occurring in a schema is allowed to depend on itself, either directly or indirectly.

Relational Database Schemata and Consistency

Relational Database Schema

A *relational database schema* is a pair $\langle \rho, \Sigma \rangle$, where ρ is a signature, and where Σ is a finite set of integrity constraints that are sentences over ρ .

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A *relational database* consists of a relational database schema $\langle \rho, \Sigma \rangle$ and an instance **DB** of its signature ρ .

The relational database is *consistent* if and only if, for any integrity constraint $\varphi \in \Sigma$ and any valuation θ :

$$\mathbf{DB}, \theta \models \varphi.$$

Outline

Unit 1: Signatures and the Relational Calculus

Unit 2: Integrity Constraints

Unit 3: **Safety and Finiteness**

Unit 4: Summary

Story so far ...

databases \Leftrightarrow *relational structures*

queries \Leftrightarrow *set comprehensions
with conditions as formulas in FOL[†]*

integrity constraints \Leftrightarrow *sentences in FOL*

[†] *first order predicate logic*

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So are there any remaining issues?

Yes!

Relational databases and RC[‡] queries should also have the following properties:

- ▶ The extension of any relation in a signature should be *finite*; and
- ▶ Queries should be **safe**: their answers should be *finite* when database instances are finite.

[†] *first order predicate logic*

[‡] *relational calculus*

Unsafe Queries

The set of answers to each of the following queries over the bibliography RDB is not finite:

Case 1 $\{(x, y) \mid x = y\}$

Case 2 $\{(pid, pub, year) \mid \text{BOOK}(pid, pub, year) \vee \text{PROCEEDINGS}(pid, year)\}$

Case 3 $\{(aname) \mid \neg \exists aid. \text{AUTHOR}(aid, aname)\}$

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Domain Independence

An RC query $\{(x_1, \dots, x_k) \mid \varphi\}$ is *domain independent* when, for any pair of instances $\mathbf{DB}_1 = (\mathbf{D}_1, =, \mathbf{R}_1, \dots, \mathbf{R}_k)$ and $\mathbf{DB}_2 = (\mathbf{D}_2, =, \mathbf{R}_1, \dots, \mathbf{R}_k)$ and any θ , $\mathbf{DB}_1, \theta \models \varphi$ if and only if $\mathbf{DB}_2, \theta \models \varphi$.

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Let (R_1, \dots, R_k) be the signature of a relational database. Answers to domain independent queries contain only values *that occur in the extension \mathbf{R}_i of any relation R_i* .

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safety \Leftrightarrow *domain independence and finite database instances*

Safety and Query Satisfiability

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Satisfiability of RC queries[†] is undecidable;

- ▶ co recursively enumerable in general, and
- ▶ recursively enumerable for finite databases.

[†] *Is there a database for which the answer is non-empty?*

Proof

Reduction from PCP (see Abiteboul *et. al.* book, p.122-126).

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Domain independence of RC queries is undecidable.

Proof

The query $\{(x, y) \mid (x = y) \wedge \varphi\}$ is not domain independent if and only if φ is satisfiable.

Range Restricted RC

Range Restricted Conditions and Queries

Given a database signature $\rho = (R_1/k_1, \dots, R_n/k_n)$, a set of variable names $\{x_1, x_2, \dots\}$ and a set of constants $\{c_1, c_2, \dots\}$, *range restricted conditions* are formulas defined by the grammar:

$$\begin{array}{l} \varphi ::= R_i(x_{i,1}, \dots, x_{i,k_i}) \\ \quad | \varphi_1 \wedge (x_i = x_j) \quad \text{where } \{x_i, x_j\} \cap \text{Fv}(\varphi_1) \neq \emptyset \text{ (case 1)} \\ \quad | x_i = c_j \\ \quad | \varphi_1 \wedge \varphi_2 \\ \quad | \exists x_i. \varphi_1 \\ \quad | \varphi_1 \vee \varphi_2 \quad \text{where } \text{Fv}(\varphi_1) = \text{Fv}(\varphi_2) \text{ (case 2)} \\ \quad | \varphi_1 \wedge \neg \varphi_2 \quad \text{where } \text{Fv}(\varphi_1) = \text{Fv}(\varphi_2) \text{ (case 3)} \end{array}$$

A *range restricted RC query* has the form $\{(x_1, \dots, x_n) \mid \varphi\}$, where $\{x_1, \dots, x_n\} = \text{Fv}(\varphi)$ and where φ is a range restricted condition.

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A query language for the relational model is *relationally complete* if the language is at least as expressive as the range restricted RC.

Range Restricted RC (cont'd)

Theorem

Every range restricted RC query is an RC query and is domain independent.

Proof Outline

Both claims follow by simple inductions on the form of a range restricted condition.
Exercise: Details.

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Theorem

Every domain independent RC query has an equivalent formulation as a range restricted RC query.

Proof Outline

1. Restrict every variable in φ to the *active domain*, and
2. express the active domain using a *unary query* over the database instance.

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Computational Properties of Query Answering

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- ▶ The **data complexity**, that is, complexity in the size of the database for a *fixed query* is
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- ▶ The **combined complexity**, that is, complexity in size of the query and the database, is
 - ⇒ in PSPACE,

Outline

Unit 1: Signatures and the Relational Calculus

Unit 2: Integrity Constraints

Unit 3: Safety and Finiteness

Unit 4: **Summary**

Query Evaluation versus Query Satisfiability

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Given an RC query $\{(x_1, \dots, x_k) \mid \varphi\}$ and a finite database instance **DB**, find all answers to the query.

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- ▶ Can be solved for fragments of RC.

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- ▶ tuples with *unknown* (but existing) values;
- ▶ incomplete relations and *open world assumption*; and
- ▶ conflicting information (e.g., from different data sources).

The Final Story

databases \Leftrightarrow *relational structures*

queries \Leftrightarrow *set comprehensions
with conditions as formulas in FOL[†]*

integrity constraints \Leftrightarrow *sentences in FOL*

safety \Leftrightarrow *range restricted RC[‡]
and finite database instances*

[†] *first order predicate logic*

[‡] *relational calculus*