Imre Simon
Universidade de São Paulo
São Paulo Brasil
is@ime.usp.br

If u and v have the same set of subwords of length m, they can be intertwinned in such a way that no new subword of length m appears.

ESCOLA DE COMPUTAÇÃO
SÃO PAULO, 15 A 27 DE JANEIRO DE 1979

São Paulo, april 30, 1981

Dear Jacques

Just for a change I am answering your two letters promptly. The card you sent me is really beautiful, thanks a lot.

① I agree with your remarks on Dominique's editorial job with one exception: I also do not like the word definition, but as you noted it is a matter of taste. Besides, I think that uniformity throughout the book must have been the reason for his supression of all our definitions. Anyway … . The constructiveness thing I put in Note A.

② Exercise 2.2. The notation $f \equiv 0\ [J_m]$ is great. I propose to put it like this.

a) $f \not\equiv 0\ [J_m] \Rightarrow |f| \geq km$

b) Let $f \in A^{km}$. Then

$f \equiv 0\ [J_m] \iff f$ has a factorization $f = f_1 f_2 \dots f_k$ with $\mathrm{alph}(f_1) = \mathrm{alph}(f_2) = \dots = \mathrm{alph}(f_k) = A$

c) Let $f, g \in A^*$, with $A = \{a, b\}$

(i) $af \equiv bg\ [J_m] \Rightarrow af \equiv bg \equiv 0\ [J_m]$

ESCOLA DE COMPUTAÇÃO
SÃO PAULO, 15 A 27 DE JANEIRO DE 1979

Let A be a k-letter alphabet. We note $f \equiv 0\ [J_m]$ if $S(m, f) = A^{\leq m}$. Show that

(a) $f \equiv 0\ [J_m] \iff fg \equiv gf \equiv 0\ [J_m]$ for all $g \in A^*$.

(b) If $k = 2$ and $A = \{a, b\}$ then for any $f, g \in A^*$

(b.1) $af \equiv bg\ [J_m] \Rightarrow af \equiv bg \equiv 0\ [J_m]$

(b.2) $ab\, f \equiv 0\ [J_m] \iff f \equiv 0\ [J_{m-1}]$

(c) $f \equiv 0\ [J_m] \Rightarrow |f| = km$

(d) Let $f \in A^{km}$. Then

$f \equiv 0\ [J_m] \iff f$ has a factorization $f = f_1 f_2 \cdots f_k$ with $\text{alph}(f_1) = \text{alph}(f_2) = \cdots = \text{alph}(f_k) = A$.

#

(3) About Exercise 2.3: I am geting excited with your ideas. I always thought that enumerating the classes is out of reach but I might have changed my mind. I have the impression that this thing is connected with the Schützenberger-Simon theorem which I call the reconstruction theory. I confess that my ideas are fuzzy and the whole thing might ... Anyway if you keep interested in ... whole.

UNDECIDABILITY AND THE TROPICAL SEMIRING A LA DANIEL KROB

Imre Simon
Universidade de São Paulo
São Paulo Brasil
is@ime.usp.br

$F_0, \ F_1, \ \ldots$

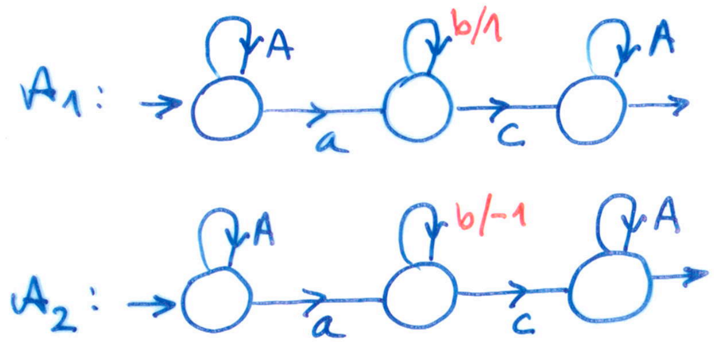
$$wF_0 = \begin{cases} 0 & \text{if } w \in U \\ -1 & \text{otherwise} \end{cases}$$

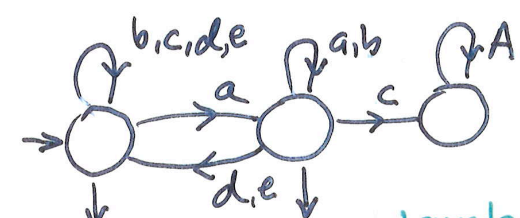
will allow to define arbitrarily wF_i on words in $\overline{U}$ provided $wF_i \leq 0$

$$F_1 = \|\mathcal{A}_1\| \odot \|\mathcal{A}_2\| \qquad sF_1 = \begin{cases} wMIN - wMAX & \text{if } s \in [\\ \leq 0 & \text{if } s \notin [\end{cases}$$

$w = s(de^+d)^{-1}$

For $s \in \overline{U}$ $\quad sF_1 = 0$ iff $s \in C$

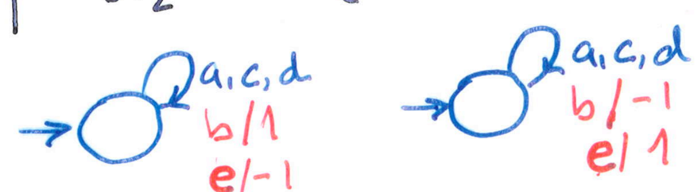




deterministic automaton assures $wF_1 = 0$ on uninteresting words. Belongs to both $\mathcal{A}_1$ & $\mathcal{A}$

$$sF_2 = -\left| |s|_b - |s|_e \right| \qquad sF_2 = \min\{ |s|_b - |s|_e, -|s|_b + |s|_e$$

$F_2 = \|\mathcal{A}\|$ $\quad \mathcal{A}$:



For $s \in C$ $\quad sF_2 = 0$ iff $s \in S$

For p composite use: $F' = F_0 \odot F_1 \odot F_2 \odot F_3 \odot F_4 \odot F_5$

with $\quad sF_3 = 0$ iff $z = p$

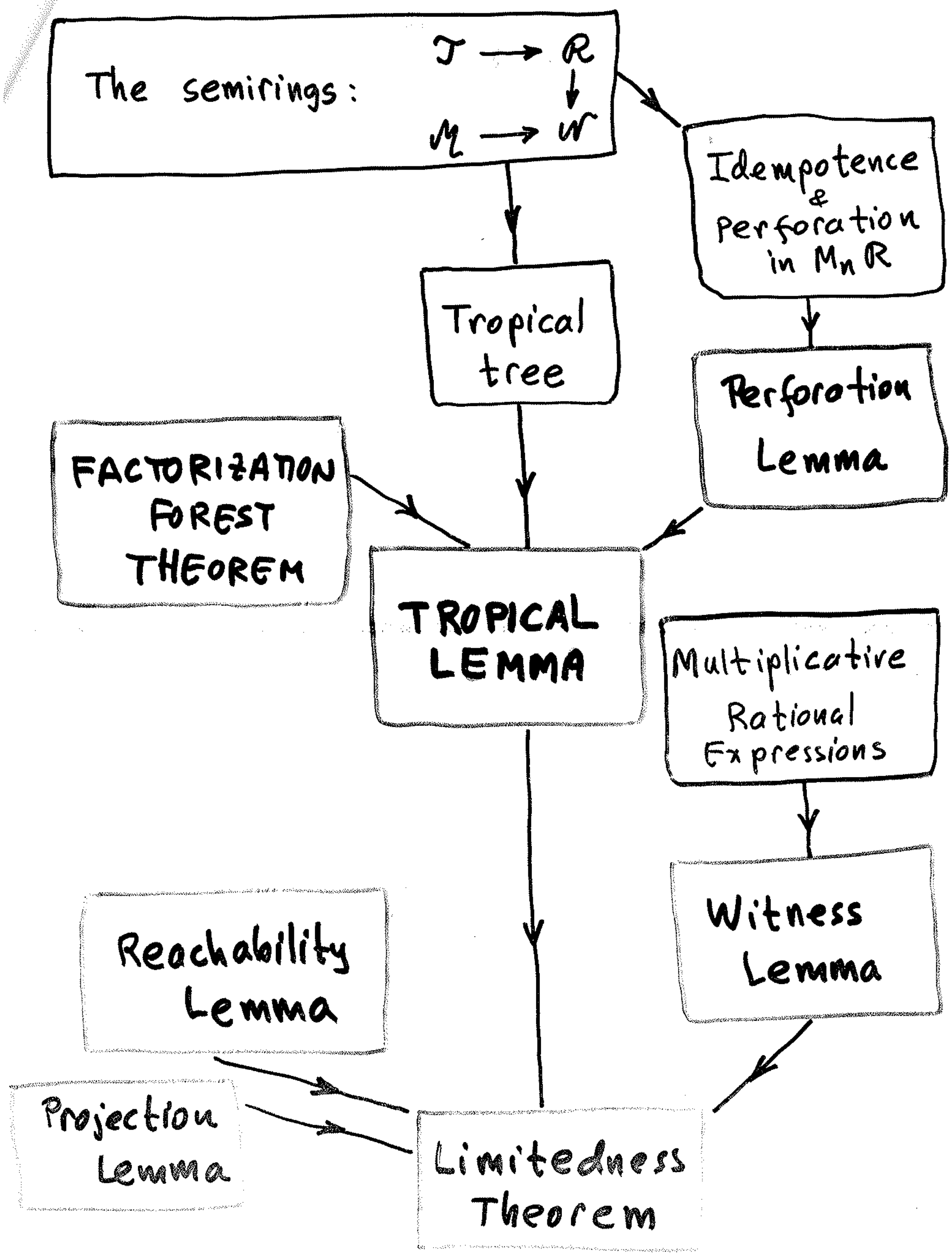
$\quad sF_4 = 0$ iff $x > 1$

$\quad sF_5 = 0$ iff $y > 1$

YET ANOTHER PROOF OF HASHIGUCHI'S LIMITEDNESS THEOREMS

Imre Simon
Universidade de São Paulo
(Brasil)

An overview


The semirings:
$\mathcal{T} \to \mathcal{R}$
$\mathcal{M} \to \mathcal{W}$
Idempotence & Perforation in $M_n \mathcal{R}$
Tropical tree
Perforation Lemma
FACTORIZATION FOREST THEOREM
TROPICAL LEMMA
Multiplicative Rational Expressions
Witness Lemma
Reachability Lemma
Projection Lemma
Limitedness Theorem

LIMITED SUBSETS OF A FREE MONOID

IMRE SIMON*

INSTITUTO DE MATEMÁTICA E ESTATÍSTICA
UNIVERSIDADE DE SÃO PAULO
05568 São Paulo, S.P., BRASIL

1. INTRODUCTION

While attending the 7th SWAT Conference in 1966, J.A.Brzozowski formulated the following problem [3]:

"Is it decidable whether for a given regular set R, $R^* = (\lambda \cup R)^m$ for some integer $m \geq 1$?"

Even though some effort has been made to clarify the above question, it has been answered only for a few particular subclasses of regular sets [10,11,12,17]. The main objective of the present paper is to give a positive answer to Brzozowski's problem. The algorithm we present

iff there exists an integer $m \geq 1$, such that $A^* = (1 \cup A)^m$. Since

$$A^* = 1 \cup A \cup A^2 \cup \ldots \cup A^n \cup \ldots, \qquad (1)$$

while

$$(1 \cup A)^m = 1 \cup A \cup A^2 \cup \ldots \cup A^m, \qquad (2)$$

it follows that A is limited iff the infinite union in (1) can be replaced by the finite union in (2), for some m. Equivalently, A is limited iff any concatenation of a finite number of elements of A can also be obtained by the concatenation of at most m elements of A.

Let us consider the set $M = \mathbb{N} \cup \infty$, where $\mathbb{N}$ is the set of all intergers $n \geq 0$. We extend to M addition, multiplication and order of $\mathbb{N}$ in the usual way ($0 \cdot \infty = \infty \cdot 0 = 0$). In the sequel we consider M as a semiring [6, p. 122] with operations

$$a \oplus b = \min\{a,b\} \text{ and } a \otimes b = a+b.$$

We will also need the semiring N with $N = \{0,1,\infty\}$ and with operations

$$a \oplus b = \min\{a,b\} \text{ and } a \otimes b = \begin{cases} a+b & \text{if } a+b \in N \\ 1 & \text{otherwise.} \end{cases}$$

Notice that N is a homomorphic image of M, and that

4. LOCALLY FINITE SEMIGROUPS

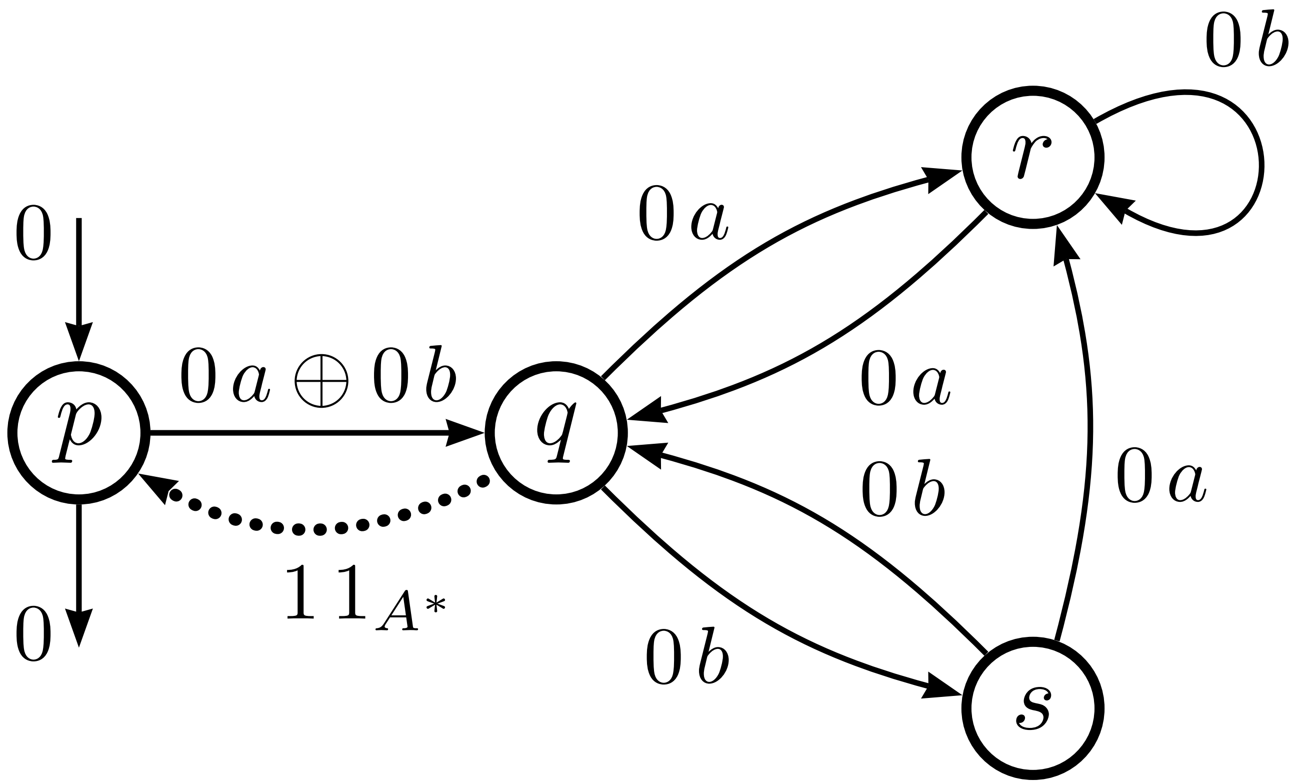
A semigroup S is *locally finite* if every finitely generated subsemigroup of S is finite. A semigroup S is *torsion* if every element in S generates a finite subsemigroup of S.

The next result is the key fact in the proofs of Theorems A and B.

THEOREM C. Every torsion subsemigroup of $M_n M$ is locally finite.

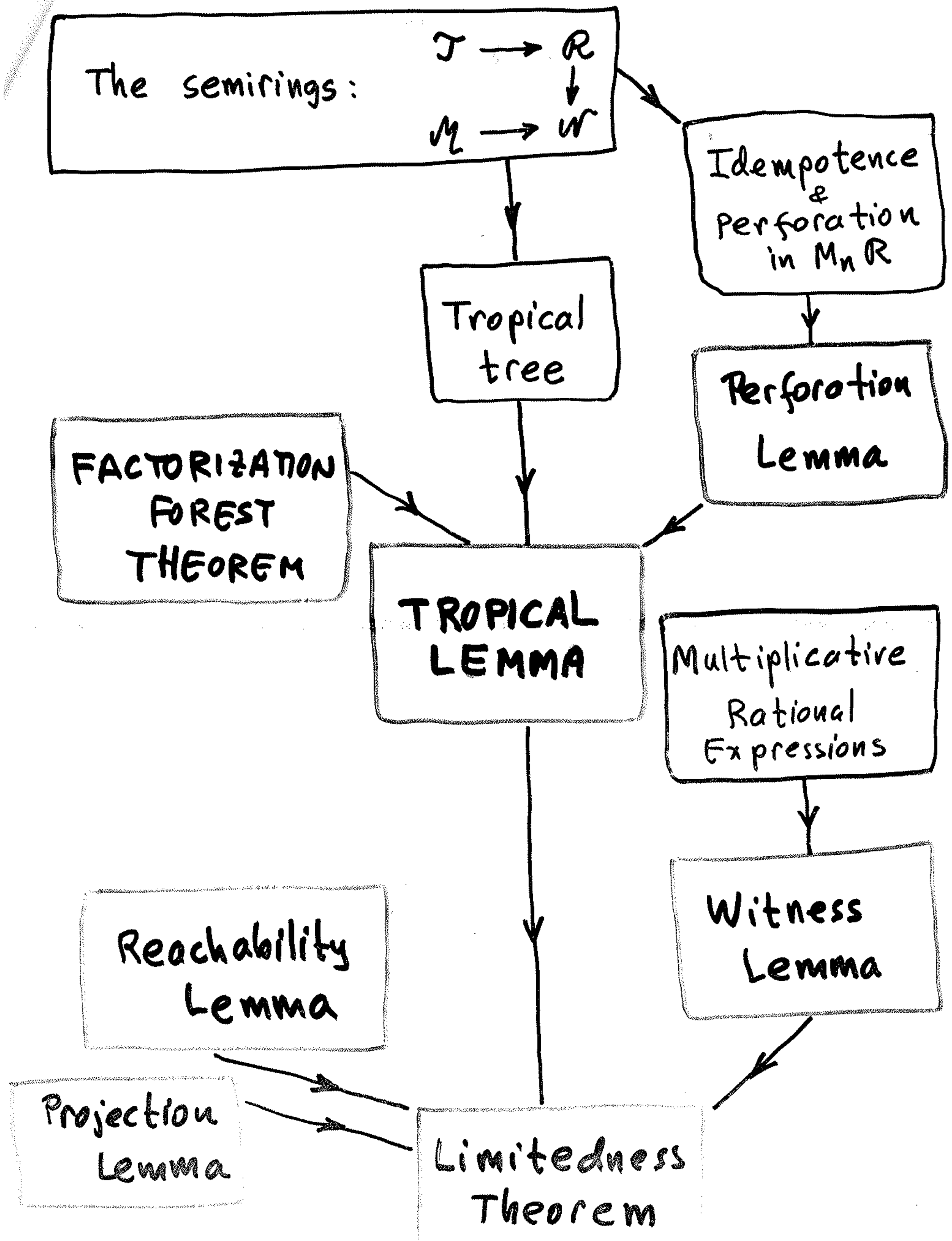
This result is interesting on its own right. Indeed, Thue's Theorem mentioned in

$$S_3 = (a+b)(b\,b + a\,b^* a + b\,a\,b^* a)^*$$



$\mathcal{M}$-automaton $\mathcal{S}_3$

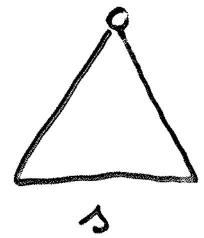
An overview



The semirings:
T → R
M → W
Idempotence & Perforation in $M_n R$
Tropical tree
Perforation Lemma
FACTORIZATION FOREST THEOREM
TROPICAL LEMMA
Multiplicative Rational Expressions
Witness Lemma
Reachability Lemma
Projection Lemma
Limitedness Theorem

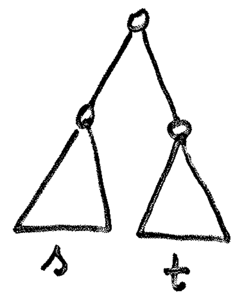
Factorization forests

Let $f: A^+ \to S$ be a morphism, with S finite

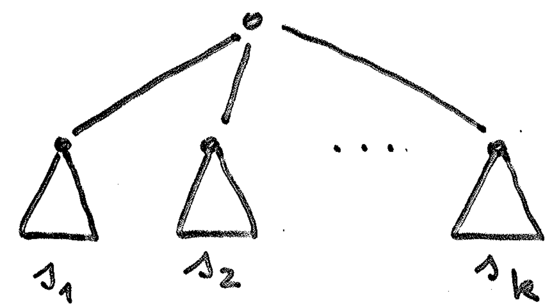


For every $s \in A^+$ we construct a tree with frond s using 2 rules:

Rule 1

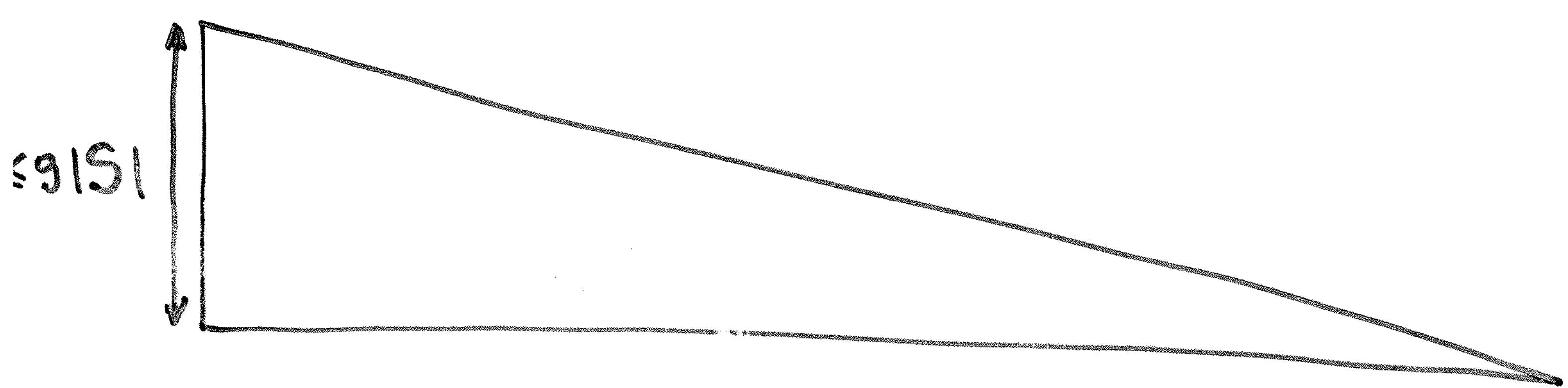


Rule 2

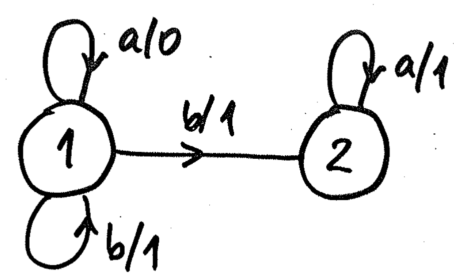


$$s_1 f = s_2 f = \cdots = s_k f = e = e^2 \in S$$

THEOREM **Every morphism $f: A^+ \to S$, S finite admits a Ramseyan factorization forest of height at most $9|S|$**

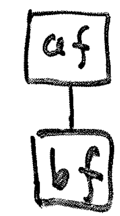


An example



$$\begin{array}{ccc} A^+ & \xrightarrow{\varphi} & M_n J \\ \downarrow & & \downarrow \psi \\ B^+ & \xrightarrow{f} & M_n \mathbb{R} \end{array}$$

$A^+\varphi\psi = \{a, b\}$



$A = \{a, b\}$ $\quad af = \begin{bmatrix} 0 & \infty \\ \infty & 1 \end{bmatrix}$ $\quad bf = \begin{bmatrix} 1 & 1 \\ \infty & \infty \end{bmatrix}$

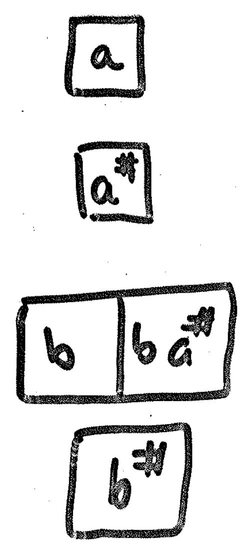
$B = \{a, b, a^\#, b^\#\}$ $\quad a^\# f = \begin{bmatrix} 0 & \infty \\ \infty & \omega \end{bmatrix}$ $\quad b^\# f = \begin{bmatrix} \omega & \omega \\ \infty & \infty \end{bmatrix}$

$S = B^+ f = \{a, b, a^\#, b^\#, ba^\#\} f$ $\quad ba^\# f = \begin{bmatrix} 1 & \omega \\ \infty & \infty \end{bmatrix}$

S: a : identity
$b^\#$: zero

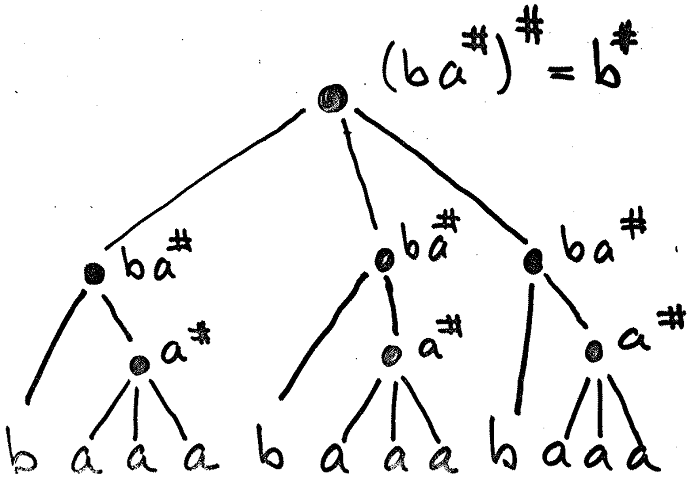
	b	$a^\#$	$ba^\#$
b	b	$ba^\#$	$ba^\#$
$a^\#$	b	$a^\#$	$ba^\#$
$ba^\#$	b	$ba^\#$	$ba^\#$

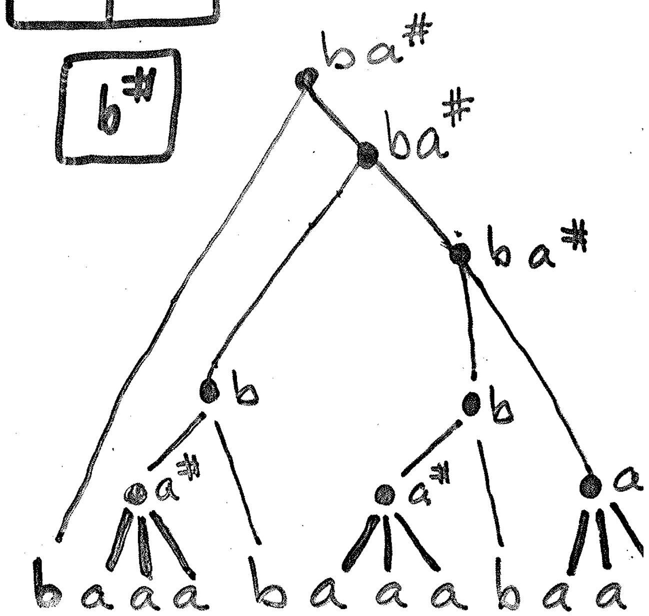
D-structure



• : discontinuous vertices

first components of labels not shown





O desbravador
da computação brasileira