A fast breast nonlinear elastography reconstruction technique using the Veronda-Westman model

Mohammadhosein Amooshahi[a] and Abbas Samani[abc]

[a]Department of Electrical & Computer Engineering, University of Western Ontario, London, ON, Canada;

[b]Department of Medical Biophysics, University of Western Ontario, London, ON, Canada;

[c]Imaging Research Laboratories, Robarts Research Institute, London, ON, Canada

ABSTRACT

A common weakness of most conventional imaging modalities is that although they can detect the presence of pathological tissues, they are incapable of classifying tumors and determining whether they are malignant. To address this major issue, elastography has been developed. This is an imaging technique that provides the spatial distribution of tissue stiffness. The main idea behind elastography is the fact that tissue pathological changes such as those associated with cancer trigger significant changes in the tissue mechanical properties. The mechanical behavior of a tissue can be described by parameters characterizing its linear or nonlinear behavior. While soft tissues demonstrate linear behavior under small strains, many clinical applications including elastography involve large strains rendering linear models inaccurate for tissue simulation. Among existing nonlinear models, the Veronda-Westman model has gained much interest because of its exponential form that is consistent with soft tissue mechanical response. However, in elastography where the spatial distribution of this model's parameters must be determined by solving an inverse problem, the exponential form poses serious challenges such as convergence and computation time. To solve the inverse problem, previous methods involved using time-demanding optimization/regularization routines. In this work, we propose a novel technique that does not involve optimization/regularization.

Keywords: Soft tissues, elastography, hyperelasticity, Veronda-Westman, inverse problem

1 INTRODUCTION

According to statistics, cancer is the second leading cause of death worldwide. There are many types of cancer humans can develop among which breast cancer is the second most common type in women [1]. Another common cancer is liver cancer, which is the fifth most common type worldwide, and has the highest mortality rate of 97% in diagnosed patients [2]. The most important factor in the treatment of all types of cancers is early detection and diagnosis. If the cancerous tissue is diagnosed at early stages, there is a greater chance of treating it with little risk to the patient's health. Currently, medical imaging is the most common way to detect cancerous lesions. While sufficient for detecting pathology, many conventional imaging modalities (e.g. CT, MR) suffer from low specificity. This means that although the presence of a tumor within the tissue can be detected, such imaging modalities provide very limited information about the type of the detected abnormality, and most importantly whether it is malignant. To address this, researchers have proposed several methods including elastography, which images tissue stiffness. This is an important development, as data indicate that various pathological tissues exhibit different stiffness characteristics.

2 THEORY

2.1 Elastography

Elastography is an imaging technique in which tissue stiffness is imaged and used to detect or classify tumors. In this work, we focus on the classification capability of elastography, as the presence of tumor can be ascertained using other conventional imaging modalities. Elastography was first introduced by J. Ophir *et al* [3]. The basic idea behind elastography is the fact that tissue pathological changes often trigger substantial stiffness changes. The

core of elastography techniques is their inverse problem of stiffness parameter reconstruction. Reconstruction techniques are based on elasticity constitutive models that are selected to model the forward problem. These are divided into linear and nonlinear (hyperelastic) models. Linear elasticity assumes that the relationship between stress and strain is linear, and uses two parameters (Young's modulus and Poison's ratio) to describe the mechanical behavior of tissue. However, given that most soft tissues exhibit nonlinear characteristics under the mechanical stimulation of elastography procedures, we employ a hyperelastic formulation. Moreover, tissue hyperelastic parameters can be used for cancer diagnosis. The constitutive relationship of incompressible hyperelastic materials is as follows:

$$S = \frac{2}{J} DEV\left[\left(\frac{\partial U}{\partial \bar{I}_1} + \bar{I}_1 \frac{\partial U}{\partial \bar{I}_2}\right)\bar{B} - \frac{\partial U}{\partial \bar{I}_2}\overline{BB}\right] \quad (1)$$

In this equation S is the deviatoric stress, DEV indicates the deviatoric part, and U is a strain energy function. Other parameters ($\bar{I}_1$, $\bar{I}_2$, $\bar{I}_3$, $\bar{B}$ and $\overline{BB}$) are functions of displacements and can be calculated using the acquired displacement data.

2.2 Veronda-Westman Model

One of the best nonlinear models in terms of providing a very close fit to typical stress-strain curves of soft tissues is the Veronda-Westman model, which was originally introduced in 1970 [4]. It has been recently used by several researchers in modeling soft tissues [5, 6, 7]. This model has an exponential form, and uses three parameters (C_1, C_2 and C_3) to describe tissue nonlinear behavior:

$$U = C_1\left[e^{C_3(\bar{I}_1 - 3)} - 1\right] + C_2(\bar{I}_2 - 3) \quad (2)$$

3 METHODS

3.1 Problem Definition

We need to solve an inverse problem where the tissue displacement data is available to reconstruct the tissue hyperelastic parameters. To form the inverse equations we substitute the Veronda-Westman energy function (Equation (2)) in Equation (1) to obtain:

$$S = \frac{2}{J} DEV\left[\left(C_1 C_3 e^{(C_3(\bar{I}_1 - 3))} - C_2\bar{I}_1\right)\bar{B} + C_2\overline{BB}\right] \quad (3)$$

This equation is nonlinear in terms of C_1 and C_3; therefore, simple matrix-based system inversion is not possible. Previous inversion techniques have used optimization and/or regularization steps for parameter reconstruction [5, 6]. The novelty of this work is the development of a new approach where no optimization or regularization is necessary. With this approach, the complexity and computation time are reduced drastically.

3.2 Novel Inversion Technique

The main idea of the proposed approach is to use an approximation to the exponential term of the Veronda-Westman energy function. Having this approximation, we use a change of variables technique, which changes the system of equations into an equivalent linear system of equations in terms of the new variables. Hence, we solve this linear system for the new variables. Finally, we determine the unknown C_1, C_2 and C_3 parameters using the obtained new variables. The whole procedure consists of three steps. In each step, one of the unknown parameters of C_2, C_3 and C_1 is reconstructed.

In the first step, we use the first three terms of Taylor's series to expand the exponential term, and then, to form the described linear system of equations. Since we used a gross approximation of the exponential term, we obtain rough estimations of C_1 and C_3 by solving this equivalent system; however, as Equation (3) is linear in terms of C_2, we find the accurate value of C_2 in this step.

In the second step, we improve the estimated C values, as we have an estimation of C_3 from the previous step, which enables us to obtain a much better approximation to the exponential term. One way to obtain such an approximation is to use the *polyfit* function in MATLAB, which uses the least squared errors measure to find the closest polynomial approximation to a given function. Fig. 1 shows the higher accuracy of the *polyfit* approximation compared to the Taylor's series expansion. Hence, once again the procedure in the first step is repeated, this time to find the accurate value of C_3.

To find C_1, Equation (3) can be solved directly for C_1 based on the obtained C_2 and C_3. This whole process takes less than a second to finish since no regularization or optimization steps are included.

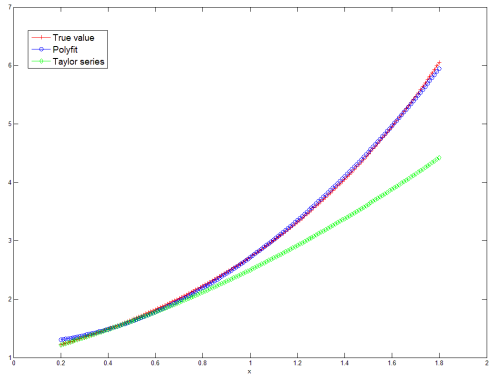

Figure 1. Comparison of two different approximations of polynomial fitting and Taylor's series

4 NUMERICAL PHANTOM VALIDATION

4.1 2-D Numerical Phantom

In this validation, we consider a breast undergoing ultrasound (US) elastography. We created a Finite Element (FE) model of a 2-D breast phantom (Fig. 2) to validate the proposed method. This phantom consists of three different layers: an elliptical inclusion, middle and outer layers which represent the tumor, fibroglandular and adipose tissues, respectively. We use ABAQUS software to perform a FE analysis where we apply 30% of compression to simulate the compression applied by a US imaging probe. To solve the inverse problem we assume that the hyperelastic parameters of the normal (adipose and fibroglandular) tissues are available [8]. Thus, we follow a constrained reconstruction procedure similar to the one presented by Mehrabian *et al* [6].

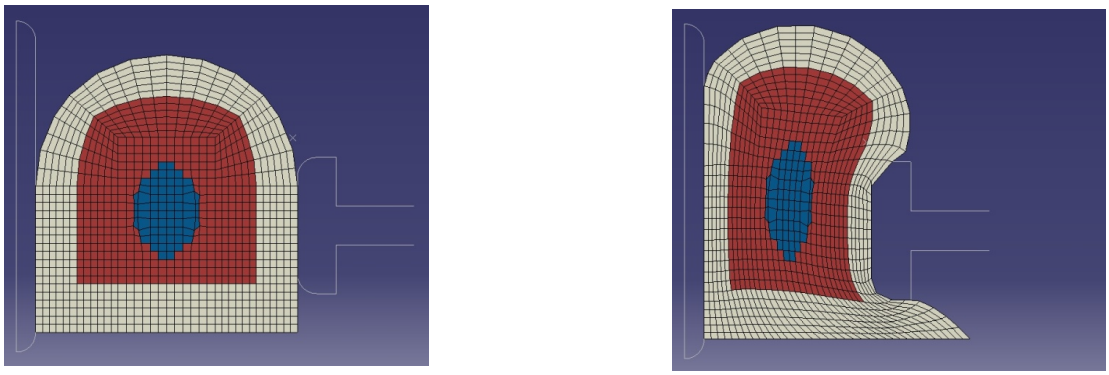

Figure 2. FE model of the phantom before applying deformation, and after applying ultrasound probe compression

4.2 3-D Numerical Phantom

In this example, a 3-D breast phantom undergoing MR elastography is studied. This phantom is similar to the 2-D counterpart. It has the same three layers (adipose and fibroglandular tissues, and a tumor). Different views of this phantom are shown in Fig. 3.

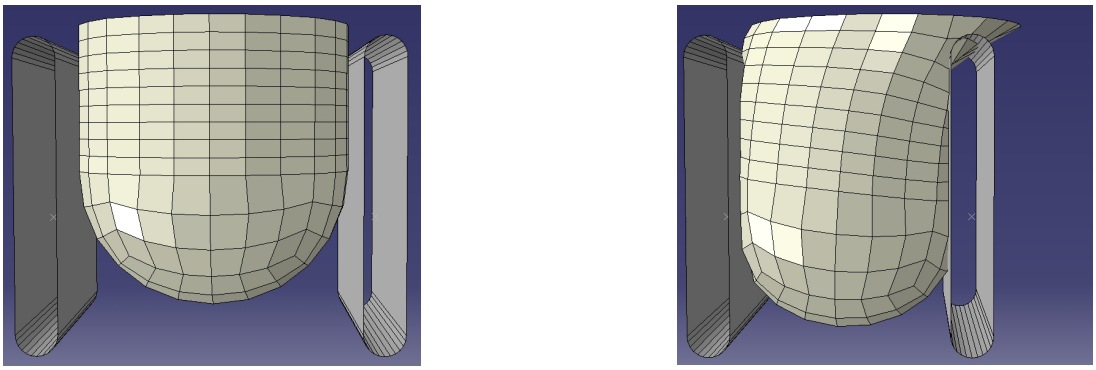

Figure 3. FE model for 3-D breast mimicking phantom in pre-compression and post-compression states

Again, we solve for the hyperelastic parameters of the tumor tissue assuming known parameters for the normal tissues. We use the same reconstruction algorithm here. The only difference is that the equations are formed considering three dimensions instead of a 2-D framework.

5 RESULTS

We tested the method on both 2-D and 3-D phantoms for a wide range of hyperelastic parameters on 100 different sets. These 100 sets of parameters were randomly selected such that they covered the entire range of hyperelastic parameters that O'Hagan [9] has reported for Veronda-Westman model of real breast tumor samples. As an indication, we have included reconstruction results for five sets of those 100 samples here. These five sets are shown in Table 1, and corresponding reconstruction results for 2-D and 3-D cases are shown in Tables 2 and 3, respectively. For the 2-D phantom, the maximum error we obtained was less than 4%, and the average error was 1.93%. For the 3-D phantom, we observed a maximum error of less than 2%, and an average error of 1.11%.

Table 1. Parameters used to construct both 2-D and 3-D phantoms

Test No.	Real Parameters (C_1 and C_2 are in kPa, C_3 is unitless)								
	Adipose tissue			Fibroglandular tissue			Tumor tissue		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
1	2.79	-1.12	1.11	5.38	-2.29	2.58	8.20	-2.15	3.39
2	3.28	-1.51	2.23	5.26	-3.46	3.13	9.37	-2.33	4.25
3	4.59	-1.60	1.29	7.67	-2.22	1.94	10.13	-3.66	2.81
4	3.81	-0.49	1.54	5.83	-3.25	2.31	11.05	-2.41	3.72
5	5.68	-1.47	1.34	8.25	-4.29	2.30	13.68	-3.72	3.38

Table 2. Reconstructed tumor parameters for the 2-D phantom

Test No.	Reconstructed Parameters (C_1 and C_2 are in kPa, C_3 is unitless)								
	C_1 (true)	C_1 (solved)	Error (%)	C_2 (true)	C_2 (solved)	Error (%)	C_3 (true)	C_3 (solved)	Error (%)
1	8.20	8.29	1.10	-2.15	-2.09	2.79	3.39	3.35	1.18
2	9.37	9.54	1.81	-2.33	-2.27	2.58	4.25	4.09	3.76
3	10.13	10.24	1.09	-3.66	-3.63	0.82	2.91	3.01	3.43
4	11.05	11.22	1.54	-2.41	-2.36	2.07	3.72	3.60	3.23
5	13.68	13.37	2.27	-3.72	-3.58	3.76	3.38	3.42	1.18

Table 3. Reconstructed tumor parameters for the 3-D phantom

Test No.	Reconstructed Parameters (C_1 and C_2 are in kPa, C_3 is unitless)								
	C_1 (true)	C_1 (solved)	Error (%)	C_2 (true)	C_2 (solved)	Error (%)	C_3 (true)	C_3 (solved)	Error (%)
1	8.20	8.06	1.71	-2.15	-2.18	1.40	3.39	3.44	1.47
2	9.37	9.27	1.07	-2.33	-2.32	0.43	4.25	4.29	0.94
3	10.13	10.01	1.85	-3.66	-3.62	1.09	2.91	2.86	1.72
4	11.05	11.03	0.18	-2.41	-2.44	1.24	3.72	3.72	0.00
5	13.68	13.54	1.02	-3.72	-3.67	1.34	3.38	3.40	0.59

6 DISCUSSION AND CONCLUSION

Elastography is a powerful technique that can be used to determine the types of breast tumors. While linear elastography lacks the capability to model soft tissues because of their nonlinear behavior, hyperelastic models are better suited for this purpose. Veonda-Westman is a hyperelastic model that has been used recently to model soft tissues, especially breast tissue. Its exponential form makes it capable of modeling nonlinear behavior of soft tissues highly accurately; however, its corresponding hyperelastic parameter reconstruction problem leads to a nonlinear system of equations.

Previously proposed techniques involve regularization and/or optimization, which are very time-consuming and are not guaranteed to converge. In this work, we presented a novel solution to this nonlinear system which is much faster and yet reasonably accurate. This solution is straight-forward, as it does not use regularization or optimization. The technique was validated using 2-D and 3-D phantom studies where its performance in reconstructing tissue hyperelastic parameters was demonstrated. These studies indicated reconstruction errors of less than 4% and 2% in the 2-D and 3-D phantoms, respectively. In terms of speed, the proposed method reduced the computation time by 1/9 compared to Gokhale's work [7], and by 1/3 compared to Mehrabian's work [8].

Future work will involve experimental validation of the proposed technique using a tissue mimicking phantom. The phantom will be constructed using PVA-C which has been used widely in modeling soft tissues because of its nonlinear behavior. The phantom's displacement data will be acquired using an ultrasound system.

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