Three Problems about Dynamic Convex Hulls

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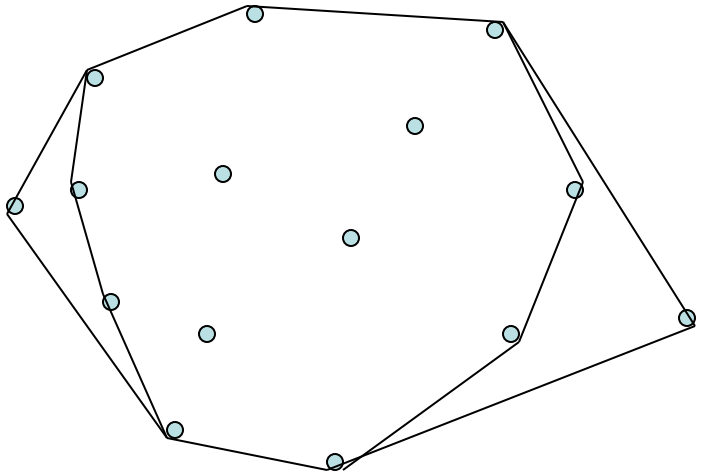
I. Dynamic 2D Convex Hulls

- Maintain point set in 2D under insertions & deletions s.t. we can answer queries about the convex hull (CH)



History

- Overmars,van Leeuwen '80 ("Hull Tree")
  $O(\log^2 n)$ update time, $O(\log n)$ query time

- C. [FOCS'99]
  $O(\log^{1+\varepsilon} n)$ update (amort.), $O(\log n)$ query

- Brodal,Jacob [SWAT'00]
  $O(\log n \log\log n)$ update (amort.), $O(\log n)$ query

- Brodal,Jacob [FOCS'02]
  $O(\log n)$ update, $O(\log n)$ query
  [current full page > 100 pages!]

Problem Solved… NOT!

- C. & Brodal,Jacob apply to "Type-A" Queries:
  - decide whether CH intersects a query line
  - extreme pt along a query direction
  - neighbor of a query vertex
  - tangent to a query pt

- But not "Type-B" Queries:
  - decide whether CH contains a query pt
  - intersect CH with a query line
  - outer/separating tangents between 2 disjoint CHs

What's Known for Type-B Queries

- C. [FOCS'99]
  $O(\log^{3/2} n)$ update (amort.), $O(\log^{3/2} n)$ query

- Open: better??

- New Result

  $O(\log^{1+\varepsilon} n)$ update (amort.), $O(\log^{1+\varepsilon} n)$ query (rand.)

  [or $O(\log n \cdot 2^{O(\text{loglog } n \text{ logloglog } n)^{1/2}})$ update, query!]

Technique

- Work in dual $\Rightarrow$ LP queries

- Based on C. [FOCS'99]:

  Logarithmic Method + Delete-Only DS

  + Interval Tree

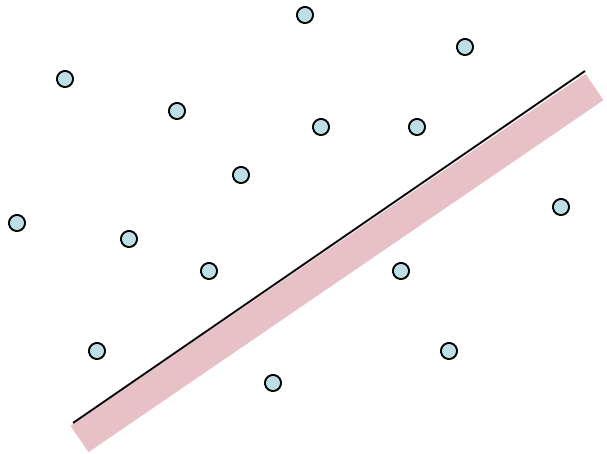
  + Bootstrapping

  NEW: + Randomized LP-type Alg'm

  (where "basis evaluation" oracle is implemented by bootstrapping)

II. Dynamic 2D Halfplane Range Reporting

- Maintain point set S in 2D under insertions & deletions s.t. we can report all k pts inside a query halfplane



[generalizes dynamic CH (k=0)…]

What's Known

- Overmars, van Leeuwen

  $\Rightarrow$ $O(\log^2 n)$ update,

  $O(\log n + k \log^2 n)$ query by k repeated deletions

  or $O(\log n + k \log n)$ query by direct recursion

- Brodal,Jacob

  $\Rightarrow$ $O(\log n)$ update, $O(\log n + k \log n)$ query

- Static case: Chazelle,Guibas,Lee '85

  $O(n)$ space, $O(\log n + k)$ query

- Agarwal,Matoušek '92:

  $O(n^{\varepsilon})$ update, $O(\log n + k)$ query

- Open: $O(\text{polylog})$ update, $O(\text{polylog } n + k)$ query??

- New Result

  $O(\log^{6+\varepsilon} n)$ update (amort., rand.), $O(\log n + k)$ query

Technique

- Based on C. [SODA'06] on dynamic 3D convex hull:

  Logarithmic Method

  + Deletion by Re-insertion
  + Hierarchy of Shallow Cuttings

  NEW: + Auxiliary DSs for Conflict Lists

  (+ bootstrap twice)

Applications

- Dynamic 3D halfspace range reporting:

  $O(\log^{6+\varepsilon} n)$ update (amort., rand.),
  $O(\log^2 n/\log\log n + k)$ query

- Dynamic 3D dominance range reporting:

  $O(\log^{6+\varepsilon} n)$ update (amort., rand.), $O(\log n + k)$ query

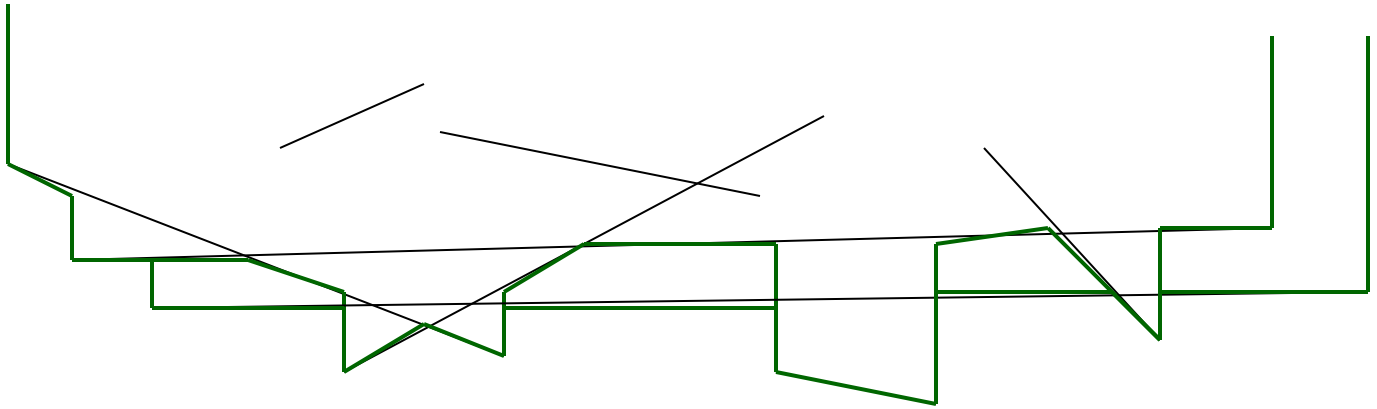
- Dynamic 3D orthogonal range reporting:

  $O(\log^{9+\varepsilon} n)$ update (amort., rand.), $O(\log n + k)$ query

III. (Semi-)Dynamic 2D Lower Envelopes of Line Segments

- Maintain set S of line segments in 2D under insertions & deletions s.t.

  we can answer queries about the lower envelope (LE)



[generalizes dynamic CH $\Leftrightarrow$ LE of lines]

What's Known

Decomposable search problem!

- O(polylog) update,query is straightforward for "Type-A" queries:
  - decide whether a query pt is below LE
  - intersect LE with vertical line
  - decide whether a query line segment is completely below LE (or partially above LE)
  - ray shooting from a pt below LE

NOT decomposable!!

- But not "Type-B" queries:
  - decide whether a query line segment is completely above LE (or partially below LE)
  - ray shooting from a pt above LE

What's Known for Type-B Queries

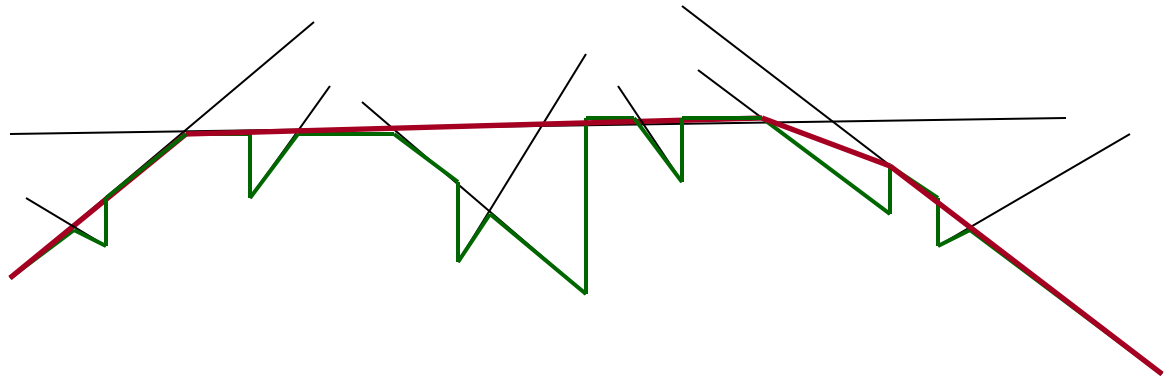
- $O^*(n^{1/2})$ update,query for insert-only
  by a method of C. [SODA'02]
- $O^*(n^{1/2})$ update,query for fully dynamic
  by dividing into $n^{1/2}$ slabs [noted by Agarwal]
- Open: better?? (say, for insert-only)
- New Result:

  $O(n^{\varepsilon})$ update (amort.), $O(\log n)$ query
  for insert-only (or "semi-online")

  [or $O(2^{O(\log n)^{1/2}})$ update, query]

Technique

- Maintain upper hull of lower envelope



- Overmars, van Leeuwen's Hull Tree
  + An Unusual Variant of Segment Tree

An Application

- Given triangulated terrain in 3D & a viewpt, can find all faces that are partially visible (or completely hidden) in

$O(n^{1+\varepsilon})$ time

Open Problems

- Dynamic CH:

  Type-B queries with $O(\log n)$ update, query??

- Dynamic halfplane range reporting:

  $O(\log n + k)$ query with better than $O(\log^6 n)$ update??

- Dynamic LE of line segments:

  $O(\text{polylog})$ update,query for insert-only??

  fully dynamic??