PWL

Optimizing Retirement Income

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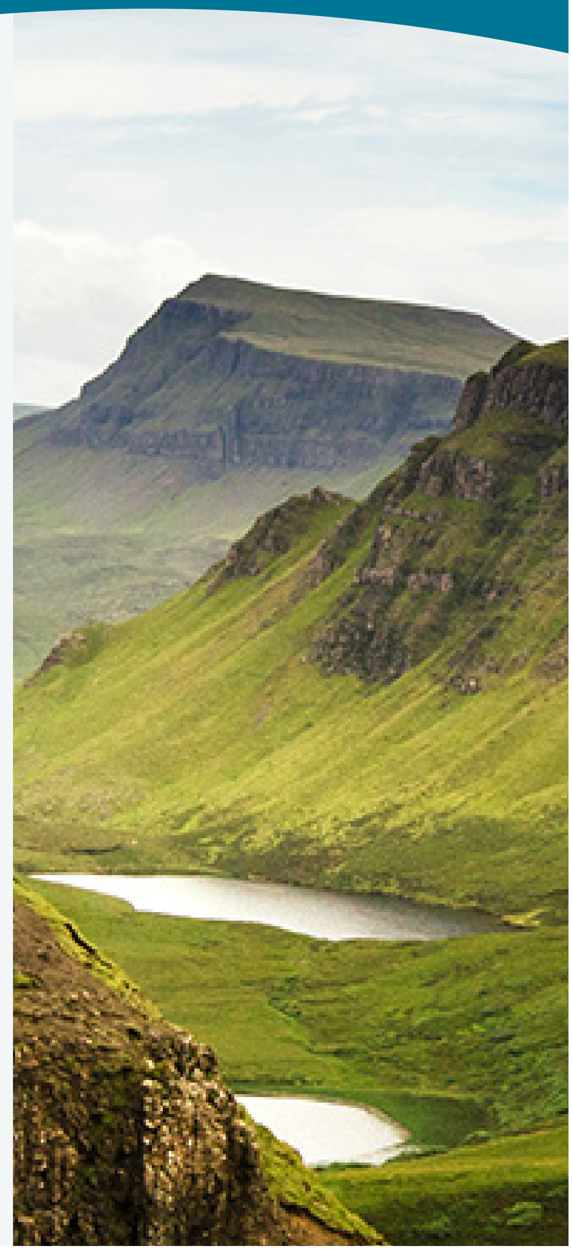

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1 Introduction

According to a recent survey (Blackrock, 2021) of American retirees most had 80% of their pre-retirement savings after almost two decades of retirement and one-third grew their assets over the course of retirement. Despite this comforting picture, there is concern (Merton, 2014) about a retirement crisis for recent and future retirees.

The Blackrock survey participants retired nearly two decades ago when the retirement landscape in the developed world was very different. Recent trends (OECD, 2020) are:

- Expected future market returns are less than historic returns. A recent report (Credit Suisse Global Investment Returns Yearbook, 2021) estimates that the real[1] annual return enjoyed by the Boomer Generation (born 1946-64) of 6.4% for a 70% equity, 30% bond portfolio will decline to 2.0% for their children, Generation Z (born 1997-2012).
- In response, pension portfolios have become riskier as bond allocations have declined. For example, Denmark's bond allocation is 27 percentage points less in 2019 than in 2009.
- A shift away from defined benefit pension (DB) plans to defined contribution savings plans. Italy, for example, closed DB plans to new members in 1993.
- Retirement portfolios are also required to last longer because retirees are living longer. The life expectancy for Canadian males(females) at age 65 increased from 13.6 years (16.9 years) in 1966 to 19.9 years (22.5 years) by 2026 (Canadian Actuaries, 2019).

Not surprisingly, recent retirees in the Blackrock survey reported increased anxiety over financial vulnerability and potential investment loss. The generation retiring now do not have the option of increasing their savings in response to lower expected returns and will be under increasing pressure to extract income more effectively from their savings. From the same survey, most of the older retirees favoured financial security over maximizing spending and relatively few retirees felt any desire to spend down their assets. Saving is a habit that can be hard to reverse and of those surveyed there was "almost no planning for systematic spend down of assets nor any asset level planning at all".

Spending down assets, or decumulation, introduces the risk of prematurely running out of money. The challenge is to consider decumulation strategies that provide more income during retirement, without increasing the risk of retirement shortfall. This is the focus of the remainder of this paper.

We first consider the basic components of any retirement income strategy: withdrawal rules, measures of risk and return and asset allocation decisions throughout retirement. We assess how familiar withdrawal asset allocation and asset allocation rules perform when assessed against shortfall risk rather than more conventional measures of risk such as volatility. We then pose the question: given a retirement goal and some reasonable spending constraints, what is the optimal solution? Optimization leads to significant improvements, illustrated by example, of how to manage retirement assets subject to decumulation.

[1] All our discussion is in real terms, net of inflation. For example, a quoted real return of 5% is equivalent to a nominal return of approximately 7% if inflation is 2%. We do this because retirees are interested in the purchasing power of their savings, which requires tracking their portfolio values in real terms. Even low rates of inflation can have a large cumulative impact on retirees over several decades.

2 What do we want, what do we really want?

We first consider the components of any retirement income strategy which involve inevitable trade-offs between what we like and what we don't like. When we are saving, or accumulating, the focus is on investment returns and the uncertainty of returns; high returns are good, highly uncertain returns are bad. Over a long period, typically two or three decades of savings, we can measure performance over a single period (a year, for example) or over the entire savings period. An analogy is driving to an important appointment. Driving quickly and overtaking other vehicles may yield short-term gains but increases the risk of not making it to your destination. In wealth terms, we can consider the in-period risk as characterised by the volatility of the portfolio. We can also consider the risk of falling short of a desired savings goal. Both can be important, and both can be useful measures of performance. We may want to assess our savings investment performance over a single year to both compare with market benchmarks but also to consider whether any losses were outside our personal risk tolerance. In addition, we want to track our cumulative progress towards a specified savings goal. Maybe we want to save $1 million to retire. Like an archer aiming at a target, we want to know how likely we are to hit the $1 million bullseye and if are going to miss, how bad a miss is it likely to be? Mostly our concern is about downside risk rather than the risk of exceeding a target which makes symmetric risk measures such as volatility inadequate.

What do retirees really want? From the survey previously cited (Blackrock, 2021) the main requirement is income certainty and the main fear is running out of money. Notice how the focus in retirement is on income rather than assets. For retirees with a defined benefit (DB) plan the separation between income and assets is complete; unless the plan itself is in jeopardy, they need not be concerned about the asset value of the plan because the plan sponsor is guaranteeing a level of income. For the rest of us, reliant on a pension from personal savings there is no such income guarantee.

Two questions dominate in retirement income planning:

- How much can I withdraw from my investments?
- How best should I invest my remaining investment assets to sustain my retirement income?

The first question requires a withdrawal rule, the second requires an investment strategy. The two questions are obviously related: how the portfolio is invested impacts the ability to sustain future withdrawals.

3 Withdrawal Rules

A well-known withdrawal strategy is the 4% rule due to Bengen (Bengen,1994). This states that a retiree holding a balanced portfolio split equally between bonds and equities can withdraw 4% of his initial wealth each year, in real terms, without running out of money. Bengen arrives at this conclusion by back testing this rule on U.S. market data showing that the retiree would never have run out of funds over any historical 30-year period.

This rule has the merit of simplicity. For example, it suggests that if you save $1 million for retirement you can withdraw $40,000 annually, indexed to inflation and be able to keep spending for 30 years. The Bengen study assumes a mix of 50% U.S. equities and 50% U.S. bonds.

The shortcomings of a fixed withdrawal rule (whether 4% or a different value) is that it is both riskier than it appears and an inefficient use of retirement resources.

The risk measurement used by Bengen considers running out of money as a failure and not running out of money as a success. Over a 30-year period running out of money in year 30 is as bad as running out of money in year 15 and a shortfall of $1 by year 30 is as bad as a shortfall of $100,000. A more refined risk measure would account for the dollar value of shortfall at the end of the retirement period. This allows a comparison between two different strategies. If the first retirement strategy has a shortfall of $100,000 and the second a shortfall of $1,000 then we assess the second strategy as better than the first, even though both are failures applying the Bengen criterion.

We introduce expected shortfall (ES) as a measure of the risk of completely depleting the assets used to generate income during the retirement period. According to this definition, If the expected shortfall is -$100,000 then this means that the average of the worst 5% of outcomes is a shortfall of $100,000. Expected shortfall quantifies the downside risk of the worst outcomes in dollars. Using expected shortfall, we can easily rank different strategies from best to worst, with the highest value preferred.

We must also consider how to sample historic data without bias. The Bengen study assembles a range of retirement sequences from the historical record. The first starts in 1926 and runs for 30 years, the next starts in 1927 and so on. In each case we could calculate the end value of the retirement portfolio and rank them from highest to lowest. To calculate the expected shortfall, as defined above, we take the worst 5% of the ending wealth and take the average. We know from the Bengen analysis that there is no instance of running out of money during any of the 30-year periods using a 4% withdrawal, so all the portfolio values at the end of the 30-year period will be greater or equal to zero. Therefore, the expected shortfall will be greater than or equal to zero. We will see that this underestimates the risk of running out of money.

Sampling historic data to test models about the future is called back testing. How this is done can have a significant impact on the conclusions. Bengen uses the historical record of market returns from 1926-1992, a period of 66 years. By using rolling historical periods, he uses some data points more than others. Any adjacent 30-year periods will have 29 years in common, any two 30-year periods beginning two years apart will have 28 years in common, etc. Because each sample period re-uses many of the same annual returns, the sample periods are not independent. This means that the variation in the annual return data is less than it would be if the returns were sampled at random. Consequently, risk measures, such as expected shortfall, are underestimated[2]. The historical record is only one example of what could have happened. To get a more accurate understanding of the range of possibilities we can consider two approaches. The first is to assume the collection of historic data points are like beads in a bag. We can replicate a random sample by taking a bead from the bag, noting the value, replacing the bead and continuing to sample. We use a variation of this approach which we identify as sampling from the historical market. An alternative approach is to construct a model of the distribution of returns based on the historic data and sample from the model. We refer to this as the synthetic market. In our research paper (Forsyth et al, 2021) we compare both the historic market and synthetic market and confirm the results are similar. For brevity, we show only the results from sampling the historical market.

Other proposed spending rules allow spending to vary as a percentage of the remaining portfolio balance. A familiar example is the RRIF withdrawal rule where the minimum withdrawal is a percentage of the balance at the end of the previous year. The percentage increases with age up to a maximum of 20% at age 95.

The portfolio can never be fully depleted using a fixed percentage withdrawal, but the income varies from year to year and can fall below the level required for a comfortable retirement.

A compromise is a withdrawal rule that allows variable spending, but the variability is spread over the remaining retirement period. An example is the Annually Recalculated Virtual Annuity (ARVA) withdrawal rule. With ARVA the amount taken out of the portfolio in any given year is based on the cash flow generated if the portfolio was used to purchase a fairly priced fixed term annuity. As the name suggests, no annuity is purchased but annuity pricing dictates the size of the withdrawal. In the case of zero interest rates over a specified retirement period the ARVA withdrawal is just the portfolio value divided by the number of retirement years remaining, which is a reasonable default.

Retirement is for an uncertain period – we want to plan for a reasonable period but to plan for the maximum (the oldest known Canadian lived to 117) would be overly conservative. Actuaries study how long we live and publish data for different age groups health, sex, and other characteristics. Using actuarial data, we assume that retirees are in the top 20% as measured by longevity. For an investor who is alive at age 80, for example, we estimate the mortality distribution of 80-year-olds and calculate the age at which 20% of the cohort are alive. Compared to the default ARVA model, this mortality boost increases spending in early retirement, while not reducing spending too precipitously during later years.

[2] An additional reason to suppose that the Bengen study underestimates *withdrawal risk* is that on occasions where there was insufficient data for a 30-year retirement period, each year of missing data was replaced with the historic average. This underestimates the risk, at least for cases where the retirement period extends past 1992.

4 Retirement Scenario

We consider the example of a 65-year-old male who has a retirement horizon of 30 years. His wealth at retirement is $1 million. He needs to withdraw at least $30,000 per year to meet essential consumption but doesn't see any benefit of withdrawing more than $80,000 per year. To provide some context, a Canadian male who has worked for 40 years in a high earning occupation can expect to retire at age 65 with government benefits of approximately $20,000 per year, giving a total income of between $50,000 and $100,000 per year.

Our retired investor withdraws cash and rebalances his portfolio at the start of each year, beginning immediately. The retirement portfolio consists of only two assets: U.S. equities and U.S. bonds. The U.S bonds are 30-day T-bills which can be considered a cash equivalent. A real-world investment portfolio would be better diversified with a wider range of assets, but this would not change our ranking of different retirement strategies. Taxes and investment fees are also not considered for the same reason.

As we will see, even if our retiree withdrew only the minimum, then because of the possibility of a sustained market downturn it is possible that he runs out of money. In this case we assume he can borrow against his house or other assets but impose a borrowing cost of 2%.

The key facts are summarized in Table 1.

Table 1

Investment horizon (years)	30
Investor	65-year old Canadian Male
Mortality Table	CPM 2014
Equity market index	U.S. CRSP cap-weighted index (real)
Bond index	U.S. 30-day T-bill (real)
Initial portfolio value	$1,000,000
Maximum annual withdrawal	$80,000
Minimum annual withdrawal	$30,000
Borrowing spread	2%
Interest rate for ARVA computation	2%
Sample period for historic data	January 1926 to December 2018

Input data for model scenario. CPM 2014 from the Canadian Institute of Actuaries.

4.1 Scenario 1: Constant equity, constant withdrawals

Table 2 presents the results when the investor makes a real withdrawal of $40,000 annually. The first column shows the equity weight as a percentage, the second column shows the expected shortfall and the third column shows the median wealth at the end of the retirement period of 30 years. Since the annual withdrawal is 4% of the initial portfolio value and is indexed to inflation, this corresponds to the 4% rule discussed above.

Table 2

Equity Weight	Expected Shortfall (ES)	Median End Wealth
0%	-$550,330	-$191,870
10%	-$461,160	-$52,680
20%	-$394,730	$113,560
30%	-$358,560	$317,350
40%	-$354,670	$562,040
50%	-$378,580	$850,230
60%	-$425,710	$1,177,310
70%	-$490,420	$1,548,450
80%	-$568,290	$1,956,860
90%	-$655,390	$2,381,870
100%	-$750,090	$2,823,110

Historical market results for constant equity over time, with annual re-balancing. Constant withdrawal of $40,000 annually. Assumes the scenario in Table 1. Statistics based on 100,000 simulations. Authors' calculations. The Expected Shortfall (ES) is the average of average of the worst 5% of outcomes.

For all equity allocations the expected shortfall is negative meaning that, in the worst 5% of cases, there is a shortfall for the investor. The example of 50% equities, 50% bonds corresponds most closely to the Bengen study and shows an expected shortfall of -$378,580, highlighting a significant shortfall risk. **Far from being risk free, a withdrawal of 4% of the initial portfolio for 30 years has a 1 in 20 chance of falling short by 38% of the initial retirement capital. This conclusion is based on historic data and takes no account of the projected decrease in expected returns noted in the introduction.**

Should we be surprised? We know from experience that equity markets can decline for years. For example, the US market declined for almost 12 years from 1998-2009. Add the additional demand of a constant annual withdrawal then it is not surprising that under a sustained market decline a portfolio can become so depleted that it cannot recover. This is often described as sequence of returns risk. At 50% equities, the median wealth at the end of retirement is $850,230 which is not far short of the initial portfolio value. Although this may be considered a windfall for any estate beneficiaries, if the intent of the portfolio was to be used for retirement then this may be considered a high cost for limited down-side protection. Is it possible to reduce the expected shortfall while using more of the retirement assets to generate income during retirement?

4.2 Scenario 2: Constant equity, variable withdrawals

We show results using our modified ARVA spending rule with a lower spending bound of $30,000 and an upper bound of $80,000 in Table 3. The extra column shows the average withdrawal over the retirement period. Compared with Table 2, the values for the expected shortfall have increased (less negative) and the median end wealth is significantly reduced. Consider the case of a 50% equity allocation. The expected shortfall has increased from -$378,580 to -$72,200 and the median end wealth has fallen to $137,970 from $850,230. The expected average withdrawal which was previously fixed at $40,000 increases to $55,200. In other words, average withdrawals have increased by 38% while shortfall risk has fallen by 80%.

Table 3

Equity Weight	Expected Shortfall (ES)	Mean expected withdrawal	Median End Wealth
0%	-$227,410	$35,790	-$13,790
10%	-$151,740	$38,530	$31,440
20%	-$98,370	$42,270	$64,710
30%	-$69,440	$46,790	$90,450
40%	-$61,860	$51,370	$111,550
50%	-$72,200	$55,200	$137,970
60%	-$99,580	$58,020	$170,370
70%	-$143,230	$59,030	$269,270
80%	-$202,740	$61,340	$493,520
90%	-$277,090	$62,230	$766,160
100%	-$362,600	$62,800	$1,069,330

Historical market results for constant equity strategies, variable withdrawals using ARVA, assuming the scenario in Table 1. Assumes the scenario in Table 1. Statistics based on 100,000 simulations. Authors' calculations.

Allowing spending to vary from year to year, within a specified range, significantly reduces the expected shortfall and boosts total spending in retirement.

Thus far we have limited attention to a constant equity allocation. A frequent suggestion is that the equity allocation should decline with age, consistent with the view that prudent investors should lower their equity exposure as their investment horizon shrinks. This has led to various suggestions for how equity allocation should change with age and there is a debate about whether the equity allocation should decline to a fixed value at retirement or continue to change through retirement. Target date funds where the equity allocation follows a curve or glide path have become enormously popular as options for defined contribution savings plans. Yet they are often poor value as there is usually a constant equity allocation strategy that will give similar results for cumulative wealth with lower fees (Forsyth el al, 2017). For that reason, it is not necessary to explore different equity allocation glide paths further.

One reason for a lack of consensus about a best investment strategy for retirees arises because studies often lack a formal definition of what is considered “best”. If we were only concerned about leaving a legacy, then a minimal income and long-term growth strategy might be the best course. Conversely if the concern is only for a constant income irrespective of how long retirement lasts then an annuity would have a lot of appeal. Different goals drive very different solutions.

In what follows we focus on some reasonable measure risk-reward that retirees seem to care about. We do not claim the objectives are universal to all retirement situations, only that once objectives and constraints are established then we can solve for an optimal solution against which we can compare other proposed strategies.

The calculations are repeated using the ARVA spending rule. The basic spending rule is modified to impose both a lower and upper bound and introduce the impact of mortality, as discussed above. The lower bound represents essential consumption. The upper bound prevents the investor drawing down too quickly when faced with early favourable returns only then to be faced with poor returns in later retirement.

4.3 Scenario 3: Variable withdrawals with optimal control of the equity allocation

A reasonable goal for retirees is to maximize annual income while minimising the risk of shortfall. To formulate this as a single target retirement goal (TRG) we define our measure of reward as the total expected withdrawals, EW, and use expected shortfall, ES as the risk measure. Informally, we can express this as:

$$\text{Target Retirement Goal} = \text{Maximum}\ (EW + \kappa ES)$$

where κ (kappa) is a weighting parameter greater than zero and is a measure how averse our retiree is to an income shortfall. We remind the reader that EW is the summation of the withdrawals *throughout* retirement and ES is the expected shortfall at the *end* of the retirement period.

We expect ES to be negative and EW to be positive so the TRG requires finding the best combination of EW and ES. On average, high total withdrawals follow from a high equity allocation but, as we have seen, a high equity allocation also generates a poor expected shortfall so there will be a trade-off. Both ES and EW are functions of the equity allocation and our optimization problem becomes one of solving for the equity allocation throughout retirement that offers the best chance of achieving the TRG.

The solution method is explained in a recent paper (Forsyth et al, 2021). In outline, we solve for the optimal equity allocation throughout the retirement period for different values of portfolio wealth. This provides a map that tells us how to update the equity allocation depending on the value at the retirement portfolio and the time to go in retirement. We then use this map to gather statistics, from sampling historic data, of how retirees fare as they experience a particular path defined by a sequence of market returns. In what follows, this solution method is called stochastic dynamic optimisation.

5 Optimal is personal

We revisit the previous scenario where we used ARVA as our withdrawal rule but this time using dynamic optimisation to calculate the best equity allocation to hit our target retirement goal. First, we look at the impact of different weightings of risk as determined by the kappa values. A low value of kappa corresponds to an investor who is more concerned about maximising income than the risk of a shortfall. She might have other assets (e.g. a house) that could be used to help fund retirement if markets performed badly or she could be willing to reduce withdrawals below the minimum ($30,000 annually, in our example). A high value of kappa would have the opposite effect.

Table 4 shows the variation of key quantities with kappa. As kappa increases the optimal solution is a less negative expected shortfall but also a lower expected withdrawal. The median wealth at the end of retirement also declines. The final column shows the average equity allocation, which also declines with increasing kappa.

Table 4

κ(kappa)	Expected Shortfall (ES)	Mean Withdrawal	Median End Wealth	Median equity allocation
0.1	-$349,500	$64,050	$258,800	46.6%
0.25	-$222,760	$63,090	$253,570	47.3%
0.4	-$136,430	$61,740	$247,420	48.2%
0.7	-$78,020	$59,810	$239,010	46.4%
1.0	-$61,230	$58,860	$230,460	45.2%
1.75	-$45,170	$56,480	$204,190	43.2%
2.5	-$40,800	$51,580	$180,320	41.6%
5.0	-$37,970	$52,260	$135,640	38.2%
10.0	-$37,340	$49,770	$101,990	33.5%

Impact of varying the aversion to income shortfall, κ. Historical market results for optimal strategies, with varying withdrawals using ARVA, assuming the scenario in Table 1. Authors' calculations.

By way of illustration, a prospective retiree who wanted an average withdrawal mid way between the maximum ($80,000) and minimum ($30,000) would settle on a kappa value of 2.5 and an expected shortfall of -$40,800 and a median equity weighting of 42%.

Thus far we have used three different investment strategies:

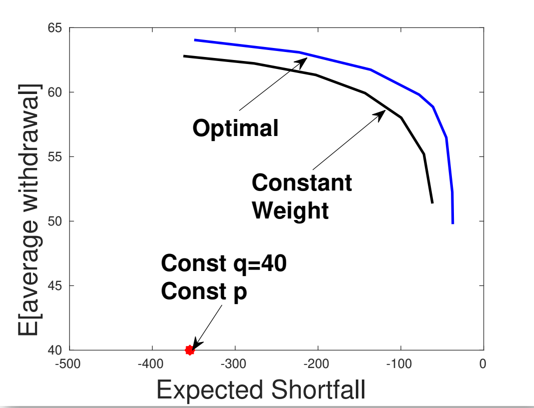
(1) The fixed withdrawal (4%) rule coupled with a constant equity allocation

(2) The ARVA withdrawal rule with a constant equity allocation

(3) The ARVA withdrawal rule using dynamic optimisation to choose the best equity allocation.

We combine the results in a single figure, Figure 1 to aid comparison.

Figure 1



The retirement income frontier with risk (ES) represented by expected shortfall and reward by expected average withdrawals (EW). See text for scenario. Dynamic optimisation (blue curve), ARVA with constant equity allocation (black curve). Constant $40,000 withdrawal with constant equity of 40% (red dot). Authors' calculations.

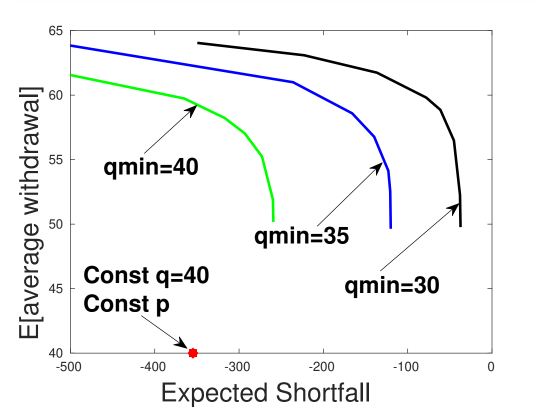
Risk, in terms of expected shortfall, is plotted on the horizontal axis while return, in terms of expected average withdrawal, is plotted on the vertical axis. The blue curve is the results from dynamic optimisation, as summarised in Table 3. There are no better combinations of ES and EW above the blue line and so we can identify this line as an efficient frontier. In other words, given the model assumptions, no other investment strategy will produce more withdrawals for a given shortfall risk. We can see values of expected shortfall greater than -$100,000 require a considerable decrease in the expected average withdrawal. The black curve below the blue curve is the ARVA withdrawal rule without dynamic optimisation and with a constant equity allocation. This is sub-optimal but for an expected shortfall below -$200,000 the reduction in expected withdrawal is approximately 5% compared to the optimal solution.

The red dot on the horizontal axis corresponds to the 4% rule, with a constant $40,000 annual withdrawal. Since all the values from Table 1 lie on the horizontal axis we only plot the best result (i.e. highest expected shortfall), which corresponds to an equity weight of 40%.

Figure 1 shows that the ARVA spending rule, even with constant equity weights, is much more efficient than a constant withdrawal strategy as represented by the red dot. **More precisely, ARVA provides about a 50% higher expected average withdrawal for the same risk, as measured by expected shortfall.** ARVA achieves this by using a higher equity allocation while limiting the expected shortfall by allowing income variability. Alternatively, we can say the cost of insisting on constant withdrawals is a less than optimal expected average withdrawal.

If the income variability from ARVA has such a significant impact it is of interest to consider increasing the minimum withdrawal. The curves for minimum annual withdrawals of $35,000 and $40,000 are plotted in Figure 2 alongside the curve from Figure 1 ($30,000 minimum annual withdrawal).

Figure 2



The retirement income frontier with risk (ES) represented by expected shortfall and return by expected average withdrawals (EW). See text for scenario. Minimum withdrawal $30,000 (black curve, qmin = 30), minimum withdrawal $35,000 (blue curve, qmin=35), minimum withdrawal $40,000 (green curve, qmin=40). Constant $40,000 withdrawal with constant equity of 40% (red dot). Authors' calculations.

As the minimum withdrawal increases the curves move down and to the left, indicating that higher minimums impose a combination of more risk (ES) and lower expected total real withdrawals (EW). Even when the minimum withdrawal is set to $40,000 the efficient frontier is well above the constant withdrawal with constant equity. **The combination of ARVA and optimal control increases total withdrawals by 25% for the same risk as using the 4% rule and constant equity.**

At first glance this might seem a surprising result and begs the question "Given that we start with the same investment capital in each case, where do the extra withdrawals come from?"

Earlier we observed that the fixed withdrawal with constant equity led to large average terminal wealth as the price for protection on the downside. Even when we impose the same minimum withdrawal as the 4% rule, the combination of variable spending and dynamic optimisation acts in concert: ARVA requires that the portfolio is spent over the retirement period so the terminal wealth is zero[3], while dynamic optimisation acts to constrain the withdrawals to limit the expected shortfall.

Dynamic optimisation imposes a counter cyclical strategy so that periods of market growth prompt a reduction in equity exposure and the risk of a large negative shortfall is reduced.

It is reasonable to ask whether allowing the optimiser to control both the equity allocation and the withdrawals would lead to greater total income. As noted above, ARVA trends to a constant payout when equity allocation is low. Without the ARVA constraint dynamic optimisation would tend to payout as little as possible in early years with a high equity allocation, switching rapidly to a high bond allocation for the remainder of the retirement.[4] This increases total withdrawals but only by the retiree having minimal income during their most active retirement years, contrary to most retirees' goals.

[3] We ignore the mortality adjustment
[4] See https://cs.uwaterloo.ca/~paforsyt/paper_optimal_DC_decum.pdf

6 Performance During Retirement

Thus far we have focused on the average withdrawals over the retirement period. We now consider the equity allocation, withdrawals, and portfolio value at different stages during retirement. These are summarised in Figure 3 for our base scenario specified in Table 1.

Figure 3(a)

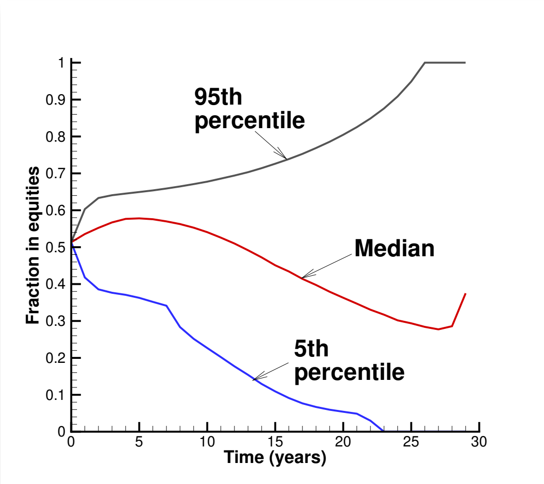


Figure 3(b)

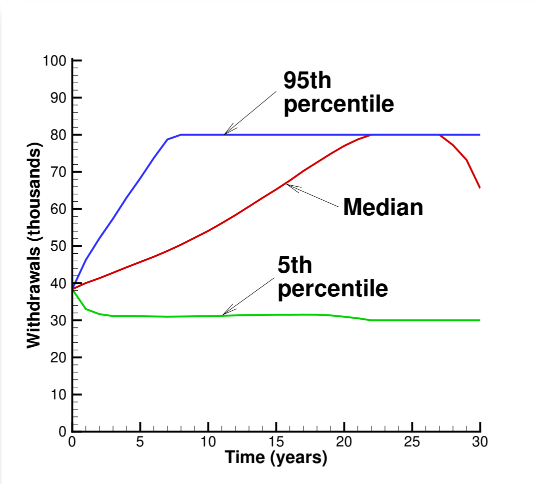
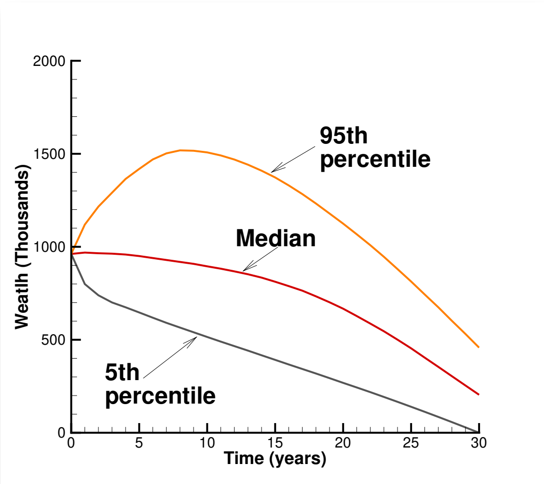


Figure 3(c)



Dynamic optimisation showing (a) fraction in equities percentiles, (b) withdrawal percentiles, (c) wealth percentiles. See text for scenario, $\kappa = 1.75$. Authors' calculations.

The results are presented as percentiles: the median is the 50% percentile. Only 5% of cases fall below the 5th percentile and only 5% lie above the 95th percentile. Figure 3(a) shows the equity allocation. In 50% of cases the equity allocation is below 60%. In 5% of cases markets have performed so strongly that after approximately 23 years all the portfolio has moved to bonds. Our algorithm forces the move to bonds as soon as the portfolio can fund future withdrawals without exposure to equities. This is a choice to de-risk. An investor could choose to continue to invest surplus cash differently to pursue estate or other objectives not included in our model.

At the other extreme, in poor markets the best chance of success is to increase the equity exposure, but the retiree may not be able to tolerate the volatility associated with high equity allocation and prefer to reduce their withdrawals.

Most of the annual withdrawals exceed $40,000, as illustrated in Figure 3(b). The chart highlights the point made by numerous researchers that the first few years can be vulnerable to sequence of returns risk. The gains from variable withdrawals and dynamic optimisation take time to accumulate with a growth in the median value of withdrawals from $40,000 to $50,000 in the first 10 years of retirement. Retirees who receive the Canada Pension Plan (CPP) have the choice of delaying payments until age 70 and benefitting from higher payouts and drawing down their retirement savings more rapidly to compensate. There is now a trade-off to consider between the benefits of delaying CPP versus the benefit of maintaining investments assets with the expectation of being able to sustain higher withdrawals.

The median wealth, Figure 3(c), declines slowly at first and then more rapidly, consistent with the earlier observation that, in most scenarios, the equity allocation falls with time, or “de-risks”, implying a steady erosion of capital in the latter years.

As noted above, dynamic optimisation solves for the optimal value of the equity allocation during retirement for a range of portfolio values. This can be visualised as a heat map as shown in Figure 4 (Figure 8.4). The map can be considered an extension of the glide paths of target date funds. No longer a pre-determined trajectory, the heat map specifies the optimal equity allocation taking account the portfolio value as the retiree progresses through retirement.

For example, suppose the retiree is 10 years into retirement and has a portfolio value of $1,000,000 then the recommended allocation to equities is 65%-70%. If the portfolio value is less than $400,000 then the only chance the retiree would have of sustaining a minimum income of $30,000 for the remaining 20 years is to invest 100% in equities.

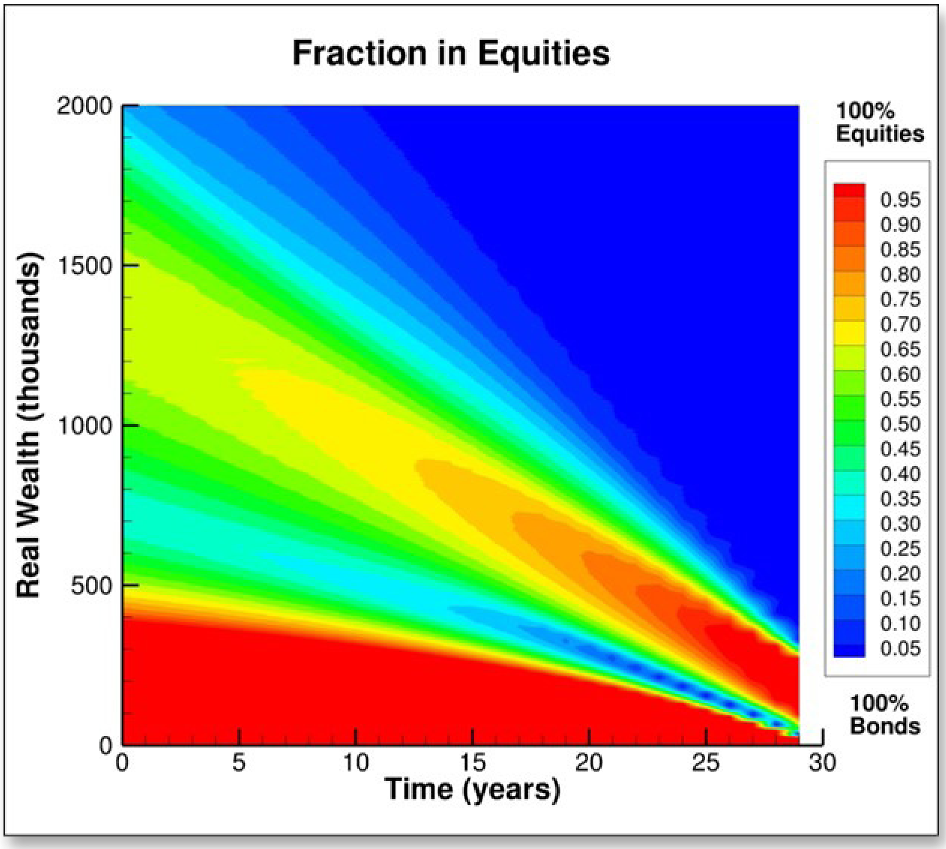


Figure 4

Heat map showing optimal equity allocation from solving the scenario identified in the text using variable withdrawals and expected shortfall, $\kappa = 1.75$. *Authors' calculations.*

At the other extreme, with high portfolio values the retiree is sufficiently wealthy that he can easily support the maximum annual income of $80,000. In this case, we force the choice that the portfolio de-risks entirely into bonds.

The overall trend of the map is from upper left to lower right, following the median wealth trend in Figure 3(c). As retirement progresses, optimal control attempts to guide the real wealth into the sweet spot between the lower blue zone and the red zone above. The lower blue zone acts as a barrier to running out of cash since the portfolio becomes very stable with a large fraction of bonds. Above the blue zone, and with little time remaining, the optimal control focuses on maximising total withdrawals by increasing the equity allocation. In practice, there may be little appetite for maximising withdrawals in late retirement as there may no longer be a desire for higher withdrawals. In most parts of the world consumption tends to reduce with age by approximately 2.75% per year above age 80 (Vettese, 2018).

7 Conclusion

Our studies suggest that retirees can safely withdraw significantly more income from their savings by allowing some flexibility in withdrawals and using dynamic optimisation to calculate the best equity allocation in response to market conditions and time left in retirement.

We employ a risk measure, expected shortfall, appropriate for retirement income planning that measures the cost of running out of money as a dollar value. We choose a spending rule, ARVA, that takes account of the portfolio value and the time to go in retirement leading to a variable withdrawal between a specified maximum and minimum. Using ARVA, the average withdrawal outperforms a constant withdrawal strategy with constant asset weights with a comparable risk, as measured by expected shortfall.

Dynamic optimisation was used to map the efficient frontier - the best combination of risk and reward as measured by expected shortfall and expected total withdrawals. Although variable withdrawals lead to major gains, the addition of dynamic optimisation lead to further improvement in total expected withdrawals, without increasing risk, as measured by expected shortfall.

Even with the minimum withdrawal set equal to the constant withdrawal, dynamic optimisation yields higher total expected withdrawals for the same expected shortfall.

Together, these results indicate that the combination of flexible withdrawals and dynamic optimisation is a powerful tool for helping retirees navigate a successful retirement. Advisors can use these tools in an increasingly challenging investment environment to add value throughout retirement by providing guidance on suitable withdrawals and equity allocation.

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