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Target Wealth: The Evolution of Target Date Funds

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ABSTRACT

Target Date Funds have become very popular with investors saving for retirement. The main feature of these funds is that investors are automatically switched from high risk to low risk assets as retirement approaches. However, our analysis brings into question the rationale behind these funds. Based on a model with parameters fitted to historical returns, and also on model independent bootstrap resampling, we find that constant proportion strategies give virtually the same results for terminal wealth at the retirement date as target date strategies. This suggests that the vast majority of Target Date Funds are serving investors poorly. However, if we allow the asset allocation strategy to adapt to the current level of the total portfolio value, significantly lower risk of terminal wealth can be achieved, at no cost to its expected value.

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1 Overview

This article analyzes the performance of Target Date Funds (TDFs) which follow *glide path* strategies in which the asset allocation depends only on the time remaining until the investor retires. We begin by calculating an optimal glide path strategy and compare the outcomes with a constant proportion equity allocation. Based on both simulated and historical markets, we find that the best possible glide path strategy offers virtually no improvement compared to constant proportion rebalancing. This confirms the observations in Westmacott (2016).

We next consider a new strategy, Target Wealth (TW). This strategy is adaptive, in that the fraction of the portfolio invested in equities depends on the achieved wealth compared to the target, as well as the time left until retirement. Significant improvements are observed with this enhanced strategy. Since the TW approach appears to be quite promising, we also discuss some practical issues associated with its implementation.

2 Saving for retirement

We first review the key objectives of saving for retirement and consider how well current offerings to retail investors are designed to meet those objectives. In particular, we consider the extent to which the increasing popularity of TDFs is related to the promise of simplicity rather than improved outcomes.

2.1 Target Date Funds: where's the beef?

Defined Benefit (DB) pension plans offered investors saving for retirement a direct connection between savings today and income tomorrow. This connection has been lost with the general decline of DB plans, accompanied by the rise of Defined Contribution (DC) plans. Participants in DC plans are left to formulate their own portfolios, coupled with the hope that time in the market will build a nest egg to provide sufficient income in retirement.

One of the weaknesses of the DC approach is that it requires knowledge about asset diversification and long term market returns, coupled with a motivation to engage in the investment process. Extreme (0% or 100% equity positions) are common. Sun Life reports that one in four plan participants still hold extreme positions.[1] TDFs tackle these behavioural shortcomings by offering a well-diversified fund of funds that starts with a high equity exposure and decreases with time until the retirement date.[2]

This changing asset allocation with age (or glide path) is the key differentiator from a conventional balanced fund with a constant proportion strategy. Vendors convey the idea that TDFs give better outcomes:

> *Don't let the markets decide when you'll retire...maximize growth opportunities early on, gradually becoming more conservative over time to protect capital and investment gains.*[3]

[1] Designed for Savings 2016: The Benchmark Report on Capital Accumulation Plans in Canada, SunLife Financial, https://www.SunLife.ca/Canada/GRS+matters/GRS+matters+articles/2016/Bright+Papers/Designed+for+Savings+Industry+Reports?vgnLocale=en_CA.

[2] According to Morningstar, assets under management for TDFs in the US reached over 700 billion (USD) at the end of 2015.

[3] See https://www.fidelity.ca/cs/Satellite/en/public/products/managed_solutions/clearpath.

The first part of this study compares TDFs with a fixed asset allocation (constant proportion) strategy and asks whether they achieve better outcomes. Our results indicate that generally, before fees, TDFs offer only a marginal improvement over a constant proportion strategy, even with the best possible glide path. In practice, since most TDFs use suboptimal glide paths, we can expect the typical TDF to perform worse than a constant proportion strategy, after fees.

2.2 Target Wealth (TW): a healthy alternative?

TDFs have some disadvantages:

- DC plan participants seek to build sufficient wealth to secure a comfortable retirement. What constitutes a comfortable retirement is subjective and depends to some extent on individual expectations. But being uncertain is not the same as being unimportant. A TDF makes no attempt to target a specific wealth outcome, but only to reach the target date with a diminished potential to grow. TDFs focus on time in the market, not the desired outcome. A target wealth (at retirement) is an important objective. However, there are tradeoffs. It is clear, for example, that a constant equity allocation of 100% would have a wide range of outcomes, after 20 or more years, from very high values to very low. Most plan participants would be willing to trade off a small possibility of achieving very large terminal wealth to avoid a significant probability of a shortfall that takes them below a comfortable (but not extravagant) level of income.
- Current wealth has no impact on future strategy with a TDF. If the plan participant has a great start and achieves her wealth target five years early, she could choose to lock in her comfortable retirement nest egg. Any surplus (i.e. not needed to fund her retirement) could be used to fund a charitable endowment, pay for the grandchildren's education, or any other purpose. Conversely if the wealth accumulation process in the early years lags expectations, then is it rational for the investor to just throw in the towel and let the TDF take her to a lower equity asset allocation and a lower expected terminal wealth at retirement?

To be more confident about securing a target retirement wealth requires a process that allows the investor to tradeoff upside potential for downside risk protection. In addition, the strategy should adapt to the current value of the portfolio (the accumulated wealth). This is the goal of the target wealth (TW) formulation. One obvious consequence is that this approach has an asset allocation which changes with time. The crucial difference is that a TDF uses a prescribed asset allocation while with the TW approach, the asset allocation is an emergent property dependent upon the investor's objectives and the investment returns over time.

A second difference is that the TW approach is a *pre-commitment* strategy: it is only effective if the investor is committed for the long term to a target level of wealth. Because TDFs have no particular goal, they can be stopped and started at will, with the general understanding that the longer the total savings period, the higher the expected final wealth.

The second part of this study proposes a TW strategy. Monte Carlo simulations and historical backtests show that this strategy is superior to glide path policies.

3 The investment problem

In the most basic TDF, there are only two possible investments: a bond index and an equity index. Given a specified target date (the anticipated retirement date of the DC plan member), the allocation to stocks and bonds is determined by a glide path. A very simple example of a glide path is

$$\text{Fraction invested in equities at time } t = p(t) = \frac{110 \text{ - your age at } t}{100}.$$

The underlying logic is appealing: investors should take on more risk when they are young (with many years to retirement) and then reduce their exposure to risk when they are older, with less time to recover from market shocks. This sounds intuitively plausible. The investment portfolio is typically rebalanced at quarterly or yearly intervals, so that the equity fraction is reset back to the glide path value $p(t)$. This idea is sufficiently attractive that TDFs have been designated as *Qualified Default Investment Alternatives* (QDIAs) in the US. If an employee has enrolled in an employer-managed DC plan, then the assets may be placed in a QDIA by default, absent any further instructions from the employee.

It is important to emphasize that the glide path $p(t)$ in the age-based example above is only a function of time. We call this type of strategy a *deterministic* glide path: it does not adapt to market conditions or the investment goals of the DC plan member.

Westmacott (2016) noted that the case for using TDFs compared to a constant equity proportion strategy appears to be rather weak, especially after fees are taken into account. We verify that conclusion here, using a parametric market model and bootstrap resampling of historical data. We then show that an optimal *adaptive* strategy (i.e. one where the asset allocation depends on the accumulated wealth) can result in the same expected final wealth as a deterministic glide path strategy, but with significantly smaller risk.

3.1 Modelling real equity returns

Using monthly US data, we construct two real (inflation adjusted) indexes: a real total return equity index and a real short term bond index. The data covers the period from *1926:1* to *2016:12*. We fit the historical returns to a parametric *jump diffusion* model. This model allows for non-normal asset returns since it represents low probability events ("fat tails") more accurately than a Gaussian (i.e. normal) distribution. The model assumes that equity returns have a Gaussian distribution in *typical* times, punctuated by large drops in equity values during periods of market turmoil. Appendix A provides further details about both the data and the jump diffusion model.

3.2 The synthetic market and the historical market

We test investment strategies using two *markets*. We refer to the market modelled by a parametric jump diffusion model, where the parameters are fitted to the entire *1926:1* to *2016:12* data, as the *synthetic market*. The *historical market* is the actual observed data set of real returns. We can view the synthetic market as a parametric model which captures the major statistical properties of the observed time series. Of course this is an imperfect representation.[4] We use the synthetic market to determine optimal strategies and carry out Monte Carlo simulations. As more of an acid test, we also study the strategies using samples from the actual historical data. We do this using *bootstrap resampling*, rather than a parametric model. If we believe that historical returns are indicative of possible future returns, then this is a model-free way to test strategies. The bootstrap approach we use is described in Appendix B.

Let W_t be the value of the investor's portfolio at time t. Given a retirement date T, the investor's final portfolio wealth is W_T. We evaluate strategies by considering characteristics of the probability distribution of W_T such as the expected value (the mean) $E[W_T]$ and the standard deviation $std[W_T]$, as for the traditional risk-reward tradeoff from mean-variance analysis. If two strategies have the same $E[W_T]$, we should prefer the one having lower risk (smaller $std[W_T]$). Note that we only consider criteria based on final wealth W_T. Some may object to this, believing that we should also be concerned with the volatility of the investment portfolio during the entire investment period. Our approach reflects the idea that investors should focus on long term goals, and not pay excessive attention to short term market fluctuations.[5]

3.3 An investment example with periodic rebalancing and regular contributions

We consider the prototypical DC investor example shown in Table 3.1. The investor's retirement is in T = 30 years. She starts off with an initial wealth of W_0 = \$0, and contributes \$10,000 per year (real) to the DC fund at the start of each year. We could make other assumptions, such as escalating the real amount contributed each year. The final results are similar to what we report here, and we make the assumption of a constant (real) contribution to keep things simple.

A common strategy is a *constant proportion* policy. For this example, we use a 60:40 equity-bond split. In other words, at each rebalancing date (yearly) we adjust the portfolio so that it has an equity fraction of 60% and a bond fraction of 40%. This is a special case of a glide path strategy with $p(t)$ = .60 at all rebalancing times.

[4] "All models are wrong: some are useful" – G. Box.

[5] Benjamin Graham, author of The *Intelligent Investor*, suggested that investors should ignore the day-to-day price swings of *Mr. Market*. Similarly, Charlie Munger (Vice-Chairman of Berkshire Hathaway) has observed that "if you are investing for 40 years in some pension fund, what difference does it make if the path from start to finish is a little more bumpy or a little different than everybody else's so long as it is all going to work out well in the end? So what if there is a little extra volatility?".

Investment horizon T	30 years
Initial investment W_0	\$0
Real investment each year	\$10,000
Rebalancing interval	1 year

Table 3.1: *Long term investment case. Cash is injected and rebalancing occurs at* t = 0,1..., 29 years*.*

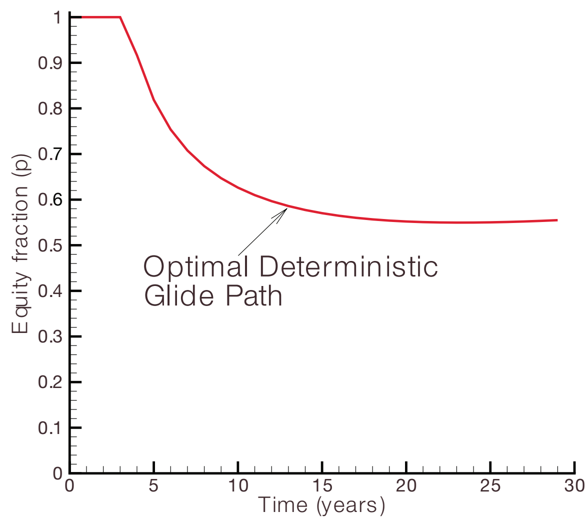


Figure 3.1: *Optimal deterministic glide path based on the synthetic market model for the example in Table 3.1. The glide path minimizes* $std[W_T]$ *with* $E[W_T]$ = \$790,000*. The parameters for the synthetic market model are determined by fitting to the entire historical data set from 1926:1 to 2016:12. Source: author calculations.*

Next we determine the optimal deterministic glide path strategy. There are clearly many ways to define *optimality*. Our approach involves finding equity fractions $p(0)$, $p(1)$,..., $p(29)$ at rebalancing times t = 0,1,..., 29 years such that

- the expected value of the terminal wealth $E[W_T]$ is the same as for the constant proportion strategy; and
- the standard deviation of the terminal wealth $std[W_T]$ is as small as possible.

We calculate the optimal equity fractions with a computational optimization method. We constrain the fractions so that $0 \leq p(t) \leq 1$ (no shorting and no leverage). The resulting optimal deterministic glide path is shown in Figure 3.1, which indicates that the constraint $p(t) \leq 1$ is active for $t \leq 3$ years.[6]

Table 3.2 compares the results for the constant proportion (p = .60) and optimal deterministic strategies. The optimal deterministic standard deviation is about 98% of the constant proportion strategy, a very small improvement. Investors may be more concerned with probability of shortfall, so we also show these results in Table 3.2. For example, the probability of achieving a final wealth $W_T <$ \$800,000 is about 63% for each strategy.

[6] Without this restriction, the strategy would use leverage to set $p(t) > 1$ during this initial phase.

			Probability of Shortfall		
Strategy	$E[W_T]$	$std[W_T]$	$W_T <$ \$500,000	$W_T <$ \$650,000	$W_T <$ \$800,000
Constant proportion	\$790,000	\$464,000	.25	.46	.63
Deterministic glide path	\$790,000	\$456,000	.24	.46	.63

Table 3.2: *Synthetic market results for the example in Table 3.1. The constant proportion strategy has* p = .60 *and the optimal deterministic glide path is from Figure 3.1. Source: author calculations.*

Based on these synthetic market results, it is possible to come up with a deterministic glide path which beats a constant proportion strategy, but not by very much. We have repeated these tests with many different synthetic market parameters. As long as the investment horizon exceeds about 20 years, the differences in performance between the optimal deterministic glide path and the constant proportion strategy with the same expected final wealth are very small.

3.4 Bootstrap resampling results

The synthetic market results above are based on fitting the historical returns to a jump diffusion model. This model assumes that monthly equity returns are statistically independent, which is questionable. To get around artifacts introduced by our modelling assumptions, we test the two strategies (glide path and constant proportion) using a *bootstrap resampling* method (see Appendix B). We emphasize that the bootstrap approach to historical backtesting has greater statistical validity compared to the commonly used rolling month technique, as in, for example, by Bengen (1994).

The bootstrap results are shown in Table 3.3. The deterministic glide path is the optimal path from the synthetic market. It is interesting to observe that the expected values for both strategies are about 5% below the results for the synthetic market values, which is quite reasonable agreement. However, the standard deviations are about 30% smaller than the synthetic market results, which suggests that serial dependence effects exist in the real data. This also suggests that, in terms of variance, the synthetic market is riskier than the actual historical market. The shortfall probabilities for both the synthetic market and the bootstrap tests are quite similar.

We show the cumulative distribution functions from the bootstrap tests in Figure 3.2. There are actually two curves here, one for the constant proportion and one for the optimal deterministic glide path, but they are almost on top of each other. The cumulative distribution function shows the probability of shortfall for all values of terminal wealth W_T. For example, we can see that the probability that $W_T <$ \$600,000 is about 30%. This plot shows that the optimal deterministic glide path is virtually identical to the constant proportion strategy in terms of the distribution of W_T.

3.5 Why are glide paths ineffective?

We suggest that the attraction of TDFs is largely a manifestation of behavioural biases due to framing. Consider the following:

Option A: An investment strategy that has a high return potential early on and a move to safety in later years.

Option B: An investment strategy that has the same expected return throughout.

or

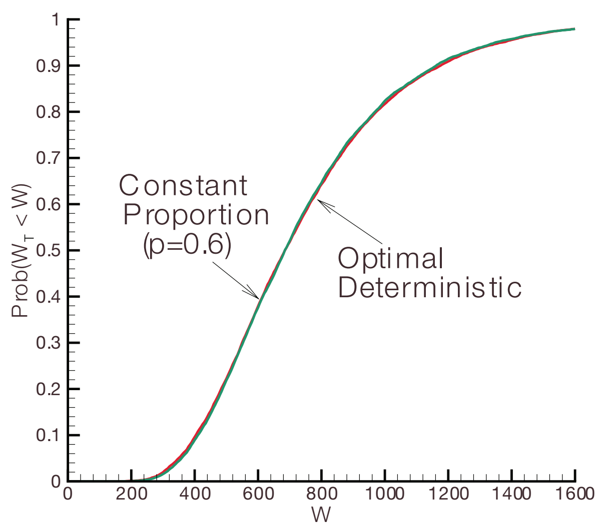
Option C: An investment strategy that has the greatest potential for losses early on, and a limited potential for gain in later years.

Option D: An investment strategy that has the same expected return throughout.

			Probability of Shortfall		
Strategy	$E[W_T]$	$std[W_T]$	$W_T <$ \$500,000	$W_T <$ \$650,000	$W_T <$ \$800,000
Constant proportion	\$745,000	\$327,000	.22	.46	.65
Deterministic glide path	\$743,000	\$320,000	.22	.46	.65

Table 3.3: *Bootstrap resampling results for the example in Table 3.1. The constant proportion strategy has p = .60 and the optimal deterministic glide path is from Figure 3.1. Results are based on historical data from 1926:1 to 2016:12 with 10,000 bootstrap resamples and a blocksize of 2 years. Source: author calculations.*

Figure 3.2: *Cumulative distribution functions of W_T for the example in Table 3.1. Units of wealth: thousand of dollars.*



The constant proportion strategy has p = .60 and the optimal deterministic glide path is from Figure 3.1. Results are based on historical data from 1926:1 to 2016:12 with 10,000 bootstrap resamples and a blocksize of 2 years. Source: author calculations.

When framed in this way, most investors would choose

- Option A over Option B;
- Option D over Option C.

However, Options A and C are simply different descriptions of the same glide path strategy, while Options B and D are the same constant proportion strategy.

The typical glide path strategy has a high allocation to equities early on in the wealth building stage. As noted by Arnott et al. (2013), this is precisely the time when the portfolio is small, so the effect of high expected returns is small as well. Conversely, during later times when the portfolio is large, there is a high allocation to bonds, with low expected returns. At the end of the day, these two effects roughly cancel out, so that we get virtually no improvement compared to a constant proportion strategy with the same $E[W_T]$.

3.6 Shall we glide?

We have repeated the above tests for different choices of the bond index (e.g. 10-year US Treasuries) and other equity indexes (such as an equally-weighted market index). We have also used different techniques to estimate the synthetic market parameters and carried out Monte Carlo simulations where we randomly perturb the market parameters. We have used various bootstrap techniques and also experimented with different glide paths. The results of all these tests are consistent. By various criteria (standard deviation, probability of shortfall), an optimal deterministic glide path achieves virtually the same result as a constant proportion strategy if both strategies are constructed to have the same expected final wealth. This confirms the conclusions of Westmacott (2016). However, most deterministic glide paths suggested in the literature do not look like the optimal path in Figure 3.1. Typical glide paths are often noticeably worse than a constant proportion strategy.[7] So, while there may be some intuitive appeal to a deterministic glide path strategy, the synthetic market tests and the bootstrap tests demonstrate that this intuition is misleading.

When faced with a choice between a deterministic glide path and a constant proportion strategy, we might as well stick with the constant proportion strategy. It is simpler, and the improvement compared with using the best possible deterministic strategy is marginal. Once fees are taken into account, the constant proportion strategy will almost always be a better choice. This might be useful information for the current investors holding TDFs.[8]

[7] Examples can be found in Dang et al. (2017), Dang and Forsyth (2016), and Forsyth and Vetzal (2017).

[8] We note that we are not the only ones to notice that TDFs based on deterministic glide paths do not seem to perform as advertised (see, e.g. Basu et al., 2011; Arnott et al., 2013; Graf, 2017).

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4 A better solution: Target Wealth

So far we have restricted attention to *deterministic* glide paths which have rebalancing strategies that depend only on time. However, if we allow our strategies to adapt to the wealth accumulated so far, we can do much better.

We can use an analogy from the basketball world here. A player following a deterministic strategy would always shoot at the same angle and velocity, no matter where he was on the court in relation to the basket. This corresponds to using a fixed asset allocation (or a glide path). The player is following a deterministic process, not focused on an end goal (sinking a basket). In contrast, a player following an adaptive strategy will shoot at different angles and velocities, depending on where he is in relation to the basket, always keeping in mind that the goal is to sink the basket.

In the financial context, let's consider a strategy whereby we allow the fraction invested in the risky asset to be a function of both time t and accumulated wealth at that time, W_t. In this case, $p = p(W_t, t)$, so that this an adaptive strategy. We constrain the strategy so that no shorting or leverage is permitted, i.e. $0 \leq p(W_t, t) \leq 1$.

We consider a target-based strategy. We choose $p(W_t, t)$ to minimize

$$E\left[(W_T - W^*)^2\right], \tag{4.1}$$

where $E[\cdot]$ denotes expected (mean) value, W_T denotes final wealth at time T, and W^* is a quantity related to target wealth (more on this below). We are finding the asset allocation strategy which minimizes the expected (mean) quadratic shortfall with respect to the quantity W^*. Note that a large shortfall is penalized more than a small shortfall. We solve this problem using an optimal stochastic control approach (Dang and Forsyth, 2015).

So what is W^*? As noted above, W^* is related to the level of target wealth. However, it is important to understand that it is *not* the same as target wealth. W^* is actually a little above the target. To achieve a particular target $E[W_T]$, we have to aim higher.[9] Returning to our basketball analogy, we always have to aim higher than the basket in order to sink the shot, knowing that the trajectory of the ball will take it lower into the basket.

Once W^* is specified, our strategy will have $E[W_T] < W^*$. But how should we pick W^*? A reasonable approach is to enforce the constraint

$$\overbrace{E[W_T]}^{\text{optimal adaptive}} = \overbrace{E[W_T]}^{\text{optimal deterministic}} \tag{4.2}$$

in the synthetic market. In other words, we select W^* so that the adaptive and the standard 60:40 deterministic strategies have the same expected final wealth.

[9] "We aim above the mark to hit the mark" – Ralph Waldo Emerson.

Note that we are using the term adaptive here in a very specific sense. We allow the equity allocation to change so as to minimize the expected squared difference between final wealth and W^*. This has nothing to do with adaptive portfolio strategies which change the asset allocation based on local properties such as price momentum, volatility or some valuation metric.

4.1 Mean-variance optimality

Our goal-based adaptive strategy looks like a reasonable idea, but how does it compare to our deterministic strategy? Recall that our deterministic strategy has the property that for a given value of $E[W_T]$, the standard deviation is as small as possible (amongst the class of deterministic strategies).

It is interesting to note that solving problem (4.1) subject to the constraint (4.2) turns out to be dynamically mean variance optimal (Li and Ng, 2000; Dang and Forsyth, 2016). Given the specified expected value of the terminal wealth, no other strategy has a smaller variance of terminal wealth. Since the standard deviation is just the square root of variance, this means that no other strategy has a smaller standard deviation.

Since we now allow $p = p(W_t, t)$, the strategy is adaptive. This is a larger class of strategies compared with the deterministic strategies $p = p(t)$. If we fix the mean value of the terminal wealth $E[W_T]$ to be the same for both deterministic and adaptive strategies, then we should expect to see that

$$\overbrace{std[W_T]}^{\text{optimal adaptive}} \leq \overbrace{std[W_T]}^{\text{optimal deterministic}} \tag{4.3}$$

In addition, the optimal adaptive strategy minimizes the expected quadratic shortfall (4.1). We actually minimize *two* measures of risk simultaneously: standard deviation (around the target) and quadratic shortfall (relative to W^*). This makes this strategy quite appealing.

4.2 Overshooting

What if we are lucky and at some point before our target date our accumulated wealth W_t is sufficient to guarantee that an investment strategy with no equity risk exposure will result in final wealth that is larger than W^*? To ensure that we penalize only losses below W^* (and not gains above W^*), we allow the investor to withdraw cash from the portfolio. We optimally de-risk at time t, using the following strategy. Suppose that

$$W_t > PV_t\,(W^* + \text{future contributions})\ , \tag{4.4}$$

where $PV_t\,(\cdot)$ indicates the present value as of time t. Then the optimal de-risking strategy is to withdraw cash in the amount of $W_t - PV_t\,(W^* + \text{future contributions})$ from the portfolio and switch the remainder of the portfolio (and all future contributions) entirely into the risk-free asset. Note that this is optimal since the quadratic shortfall is zero. We call the amount withdrawn a *surplus cash flow*.

4.3 You are lucky: don't take on any more risk

There is a simple logic behind this strategy. If an investor is fortunate enough to be able to hit W^* (which is larger than her goal – recall that to hit the mark it is necessary to aim above it) simply by investing in bonds, then she should *cash out her chips* and not take on any more risk. Of course, this requires investors to stick with their original target. The investor must *pre-commit* to being satisfied with the initial real target wealth.

Some would contend that this is difficult for most investors to do. This may be true, but we argue that the pre-committed target approach is an effective way to manage retirement investments.[10] One interpretation is that the reason the adaptive strategy is so successful is that we are taking advantage of the fact that most investors will have difficulty pre-committing to it.[11]

What should be done with the surplus cash? Early retirement is one option, although the additional years of retirement would have to be funded. Other possibilities would include a nice vacation, an enhanced bequest, or a charitable donation. An investor could even choose to invest all of the surplus in equities, since her retirement wealth goal has already been reached. For the purposes of discussion, however, we will assume that the surplus cash is withdrawn from the investment portfolio and invested in a risk-free asset.

4.4 Synthetic market: deterministic vs. adaptive

We reconsider the problem in Table 3.1, with the synthetic market parameters determined from the historical data over *1926:1 to 2016:12*. The results are given in Table 4.1. For the same $E[W_T]$, the adaptive strategy has a standard deviation that is about 50% lower. The probabilities of shortfall at various values of W_T are also significantly smaller.

Another interesting statistic is that the median final wealth for the deterministic strategy is $681,000, while the median final wealth for the adaptive strategy is $893,000. The optimal adaptive strategy skews the distribution so that the median is always above the mean. In contrast, glide path strategies usually have the opposite skew, with the median outcome being below the mean. As a consequence, half of the final wealth outcomes are larger than the mean using the adaptive strategy, whereas half of the outcomes are less than the mean using the glide path strategy.

[10] Perhaps a major problem in personal investing is that most investors always think that having more is better than less, even if a reasonable level of wealth has been amassed. We argue that this can be counterproductive when saving for retirement.

[11] If it was easy for investors to pre-commit, many would and we would not have as favourable a tradeoff between risk and anticipated reward.

4.5 Bootstrap resampling: deterministic vs. adaptive

We carry out a similar set of tests using bootstrap resampling from the actual historical market returns. The *optimal* strategies are those determined in the synthetic market. In other words, we use the synthetic market to determine our optimal strategy, but we test using the historical returns. The results are shown in Table 4.2. Again, the adaptive strategy appears to be superior to the deterministic strategy. In particular, the adaptive strategy has about a 35% chance of ending up with less than $800,000, while the corresponding chance for the glide path strategy is 65%. The median final wealth for the adaptive strategy is $869,000, compared to $682,000 for the deterministic strategy.

				Probability of Shortfall	
Strategy	$E[W_T]$	Median W_T	$std[W_T]$	$W_T <$ $650,000	$W_T <$ $800,000
Deterministic glide path	$790,000	$681,000	$456,000	.46	.63
Optimal adaptive	$790,000	$893,000	$215,000	.22	.34

Table 4.1: *Synthetic market results for the example in Table 3.1. The optimal deterministic glide path is from Figure 3.1. The optimal adaptive strategy sets W^* to give the same $E[W_T]$ as the glide path in the synthetic market. Source: author calculations.*

				Probability of Shortfall	
Strategy	$E[W_T]$	Median W_T	$std[W_T]$	$W_T <$ $650,000	$W_T <$ $800,000
Deterministic glide path	$743,000	$682,000	$320,000	.46	.65
Optimal adaptive	$791,000	$869,000	$192,000	.20	.35

Table 4.2: *Bootstrap resampling results for the example in Table 3.1. The optimal deterministic glide path is from Figure 3.1. The optimal adaptive strategy sets W^* to give the same $E[W_T]$ as the glide path in the synthetic market. Results are based on historical data from 1926:1 to 2016:12 with 10,000 bootstrap resamples and a blocksize of 2 years. For the adaptive strategy, the expected surplus cash flow is $23,000. Source: author calculations.*

4.6 The optimal adaptive strategy

To see how the adaptive strategy works, we plot the median fraction invested in stocks in Figure 4.1(b). For comparison, we also show the deterministic strategy (from Figure 3.1) in Figure 4.1(a). The median adaptive strategy maintains a high allocation to stocks for longer than the deterministic strategy, but de-risks faster and further as we near retirement. In Figure 4.1(b), we can see that in some cases (the upper quartile) the adaptive strategy may require a long term commitment to 100% equity exposure. During that time, it is possible that the cumulative wealth of a saver using the adaptive strategy may lag that of a fellow saver following a constant 60% equity allocation.

The reward for this stoicism is apparent in Figure 4.2, which shows the cumulative distribution functions of terminal wealth for both cases. If we are primarily interested in probability of shortfall, it is useful to consider the cumulative distribution function which includes the surplus cash flow (after all, this is a benefit of the adaptive strategy).

Figure 4.2 shows that the adaptive strategy produces smaller expected shortfalls for \$360,000 $\leq W_T \leq$ \$1,000,000. The probability of very large gains is higher for the deterministic strategy compared to the adaptive strategy. However, this is the price we pay for lowering the risk of shortfall relative to our goal. There is also a small probability that the deterministic strategy will outperform the adaptive strategy at the left tail of the plot where $W_T <$ \$360,000. A related statistic here is the conditional tail expectation (CTE). The 5% CTE is the mean of the worst 5% of the outcomes. The 5% CTE for the adaptive strategy is \$270,000, while the CTE for the glide path is \$310,000.

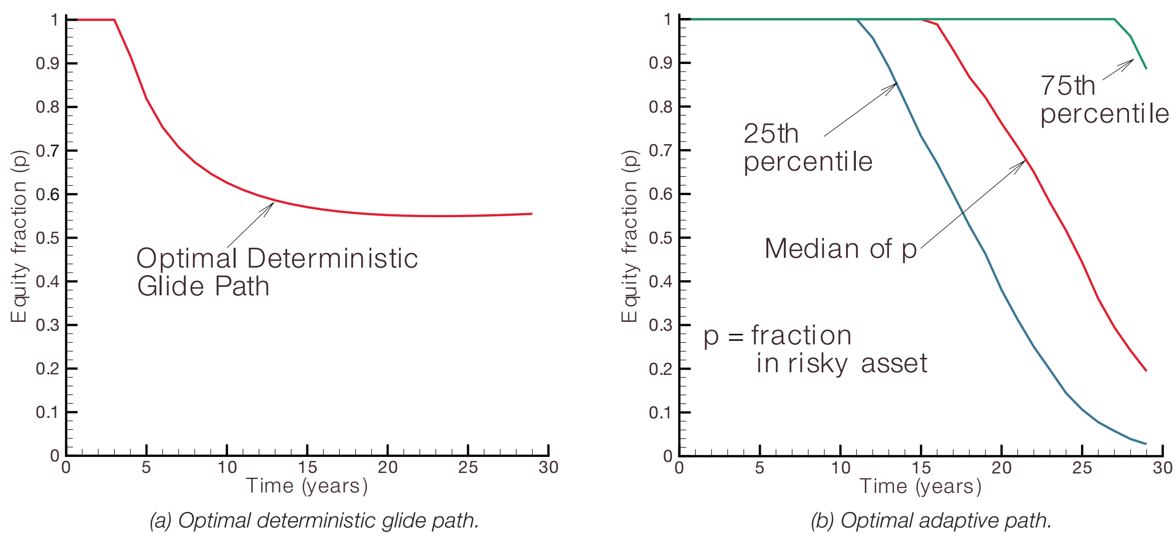

Figure 4.1: *Optimal paths: deterministic and adaptive. For the adaptive case, we show the median value of the fraction p invested in the risky asset and its 25th and 75th percentiles. The adaptive case shown is based on bootstrap resampling. Source: author calculations.*

A detailed examination of the results reveals that the left tail underperformance of the adaptive strategy occurs on resampled paths where we sample from the 1930s several times.[12] In other words, we have a resampled path where the market trends downwards over the entire 30 year investment horizon. In this case, the optimal adaptive strategy will maintain a very high allocation to equities, over a much longer period than the glide path. Of course, if we knew that this was happening, we would never invest in stocks. The problem is that nobody can reliably predict this.

5 Implementing an adaptive strategy

The statistics for using an optimal adaptive TW strategy are impressive. A useful measure of success can be phrased as *"What is the probability of getting to within 10% of the (expected) wealth target?"* Using the bootstrap tests, we find that the probability of achieving this result is about 45% for the deterministic glide path compared to 75% for the optimal adaptive strategy.

Another way to look at this is to imagine two pools of DC investors, with each pool having 1000 participants. One pool uses the deterministic glide path strategy, and the other pool uses the adaptive strategy. We choose a random month to start in the last 90 years, and pick one investor from each pool. Both of these investors accumulate wealth over the subsequent 30 years, using the glide path or adaptive TW strategy. During those 30 years, the market follows the bootstrapped historical returns. We repeat this process until we exhaust both pools. On average, 750 of the TW investors would reach their wealth goals, compared to just 450 of the glide path investors.

Of course there is no free lunch here. These gains in efficiency come at the cost of pre-commitment and the use of a contrarian investment strategy: the TW strategy can entail increasing the exposure to equity market risk after declines in portfolio wealth, at a time when large numbers of investors would be fleeing for safety. Applying these tactics generates the rewards for long term investors. This requires discipline over a very long term. Investor education is required to understand the value of pre-commitment and the contrarian nature of the strategy.

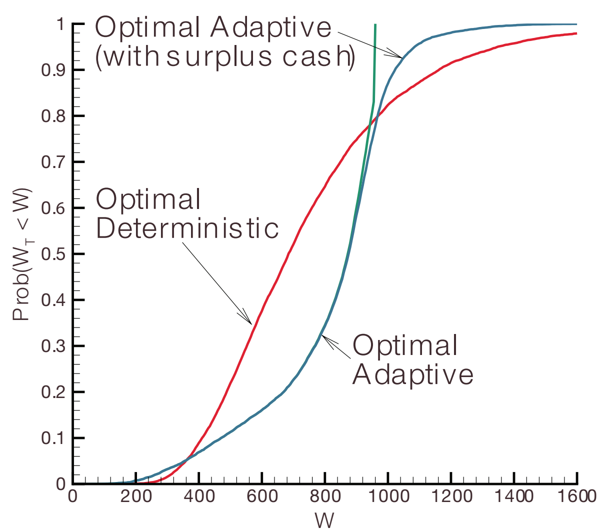

Figure 4.2: *Cumulative distribution functions of W_T for the example in Table 3.1. Units of wealth: thousands of dollars. The optimal deterministic and optimal adaptive strategies are from Figure 4.1. Results are based on historical data from 1926:1 to 2016:12 with 10,000 bootstrap resamples and a blocksize of 2 years. Source: author calculations.*

[12] Since we sample with replacement, a 30 year resampled path can sample several times from any decade.

Some additional practical issues related to the implementation of TW strategies include:

- A common criticism of TDFs is their *one size fits all* approach to investing. With the TW methodology, each DC plan participant would have their own personalized asset allocation. Even participants with the same retirement date could have different time in the market, or different savings rates, and hence different accumulated wealth. Unexpected events (receiving an inheritance, skipping a contribution to pay off a mortgage) would be automatically reflected in the strategy.

- One could envisage a range of funds with, say 5% increments in equity allocation and participants migrated from one to another, in response to each investor's individualized strategy. Such balanced funds already exist for globally diversified fixed income and equities.

- TW options would ride alongside fixed allocation funds. They would initially be more appealing to younger participants with 20 or more years to go.

6 Conclusion: to glide or not to glide?

We have looked at only one adaptive strategy, but there are many other possibilities. We argue that the adaptive strategy considered here is especially appealing because it simultaneously minimizes two measures of risk: expected quadratic shortfall (relative to $W*$) and standard deviation (around target wealth). This class of strategies is essentially contrarian, i.e. buy equities when stock prices fall, sell when they rise (provided that the investor has not been sufficiently lucky to have completely de-risked). This is also known as a concave strategy.

An essential feature of our strategy is the rules based de-risking which occurs as accumulated wealth increases. A different objective function such as minimizing the CTE can result in a strategy which sells when the market falls (to minimize the tail risk) and buys when the market rises (i.e. a momentum strategy). A common example of this sort of convex strategy is Constant Proportion Portfolio Insurance. These types of strategies increase risk as accumulated wealth increases.

We emphasize once again the considerable performance gains which can be obtained using adaptive strategies. Both Monte Carlo simulations and historical backtest simulations are consistent: compared with a glide path strategy, adaptive TW strategies reduce shortfall probabilities significantly, with the added bonus of a rules-based de-risking policy. Restricting attention to deterministic glide paths is clearly suboptimal. We can do much better by considering adaptive strategies. Since the vast majority of TDFs use deterministic strategies, they do not seem to be serving investors well.

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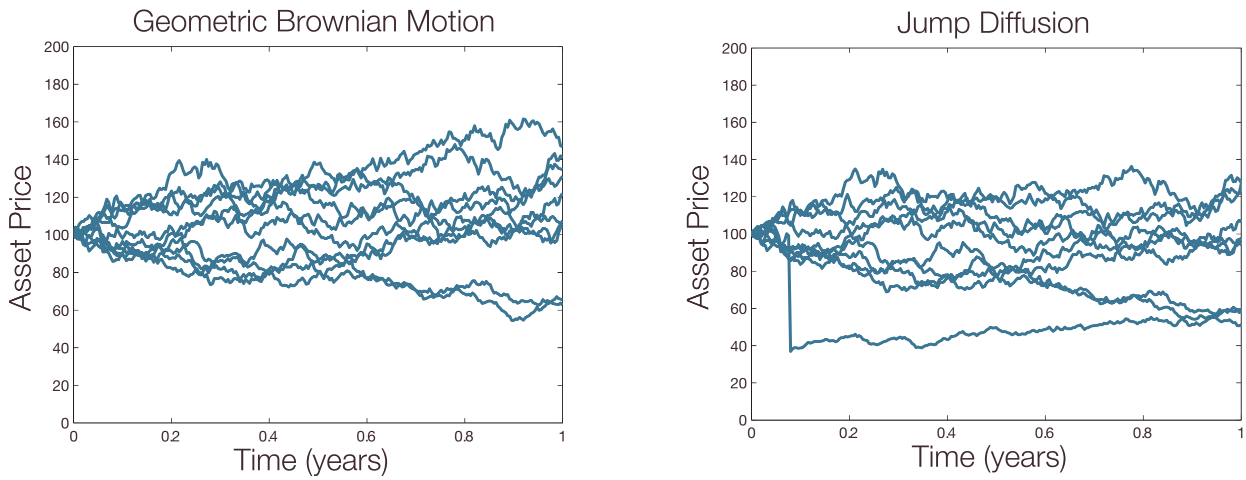


(a) Geometric Brownian Motion.

(b) Jump diusion.

Figure A.1: *Ten Monte Carlo realizations of possible random paths. For the Geometric Brownian Motion case, the expected return is 10 per cent per year, with a volatility of 20 per cent. For the jump diffusion case, the expected return and volatility are the same as for the GBM case, but in any given year there is a 10 per cent probability of a market crash. We can see this in Figure A.1(b): in any given year, we can expect a crash on one of the ten simulated paths. Source: author calculations.*

Appendices

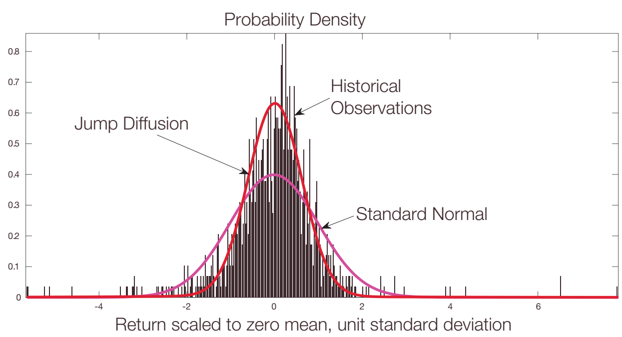
A Data and models of equity returns

The data used in this work was obtained from Dimensional Returns 2.0 under licence from Dimensional Fund Advisors Canada. In particular, we use the Center for Research in Security Prices Deciles (1-10) index. This is a total return capitalization-weighted index of US stocks. We also use one month Treasury bill returns for the risk-free asset. Both the equity returns and the Treasury bill returns are in nominal terms, so we adjust them for inflation by using the US CPI index. We use real indexes since long term retirement saving should be attempting to achieve real (not nominal) wealth goals. All of the data used was at the monthly frequency, with a sample period of 1926:1 to 2016:12.

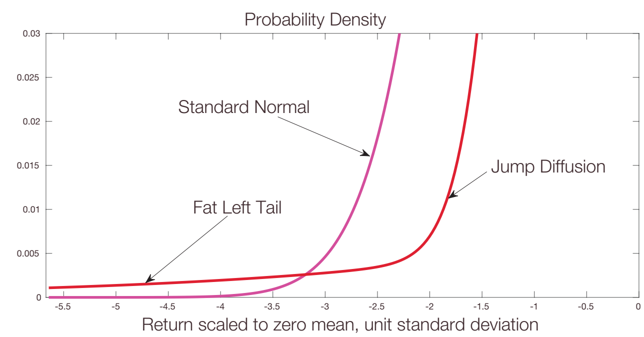
A common assumption is that logarithmic equity returns follow a normal (or Gaussian) distribution. This is embedded in the Black-Scholes option pricing model, for instance. It is generally referred to as the Geometric Brownian Motion (GBM) model. It has two parameters, one governing the expected level of returns and one describing the volatility or uncertainty associated with returns. However, this model does not account for the fact that market returns are only approximately Gaussian, and that markets tend to move sharply downward much more often than would be likely under GBM. This was all too apparent during the financial crisis of 2008.

We can augment the GBM model using a *jump diffusion* model. This model assumes that equity returns have a Gaussian distribution during usual market conditions. However, in times of market turmoil, the index exhibits abrupt large movements in the form of non-Gaussian jumps. The jump diffusion model uses additional parameters to describe the uncertain timing and size of these sudden sharp changes. Some Monte Carlo simulations of GBM (Gaussian returns) and jump diffusion models are shown in Figure A.1.

Figure A.2(a) shows a histogram of the logarithmic monthly returns from the real total return CRSP equity index, scaled to unit standard deviation and zero mean. We superimpose a standard normal (Gaussian) density onto this histogram. The plot shows that the empirical data has a higher peak and the fat left tail of the historical than a normal distribution, consistent with previous empirical findings for virtually all financial time series. The *fat left tails* of the historical density function can be attributed to large downward equity price movements which are not well modelled assuming normally distributed returns. From a long term investment perspective, it is advisable to take into account these sudden downward price movements.



(a) Probability densities.



(b) Zoom of Figure A.2(a), showing the fat left tail.

Figure A.2: *Probability density of log returns for real CRSP Deciles (1-10) index. Monthly data, 1926:1 - 2016:12, scaled to unit standard deviation and zero mean. Standard normal density and fitted jump diffusion model also shown. Source: author calculations and CRSP data from Dimensional Returns 2.0 under licence from Dimensional Fund Advisors Canada.*

We fit the data over the entire historical period using a jump diffusion model. The fitted distribution is shown in Figure A.2(a). To show the fat left tail of the jump diffusion model, we have zoomed in on a portion of the fitted distributions in Figure A.2(b). There is also a fat right tail as well, but this represents unexpected gains, which are not as worrisome as unexpected losses.

Another popular method to account for non-Gaussian returns is to use a stochastic volatility model. However, tests have shown that stochastic volatility has a negligible effect for long term investors (Ma and Forsyth, 2016). This is essentially because volatility reverts to average levels quite quickly in this type of model, so random changes in volatility have little consequence over the long haul.

B Bootstrap resampling

Bootstrap resampling has become the method of choice for historical backtesting (see, e.g. Dichtl et al., 2016). Suppose our investment horizon is T years (T = 30 in our case). Each bootstrap path is determined as follows. We divide the total investment horizon T into k blocks of size b years, so that $T = kb$. We then select k blocks at random (with replacement) from the historical dataset. Each block starts at a random month. We then concatenate these blocks to form a single path. We repeat this procedure 10,000 times and generate statistics based on this resampling method.

In order to reduce the end effects in concatenating blocks, we use the stationary block bootstrap method (Politis and Romano, 1994). We randomly select blocks with a specified expected block size. Politis and White (2004) suggest that the block sizes should be selected from a geometric distribution.

We use a paired resampling technique, whereby we simultaneously draw the blocks from the historical bond and stock index data. This preserves the correlation between these asset classes. In addition, serial dependence in the real data will show up in the bootstrap resampling. Based on some econometric criteria, we use an expected blocksize of 2 years. We have experimented with expected blocksizes ranging from .50 to 10 years, and the results are qualitatively similar.

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