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The effect of false-name bids in combinatorial auctions: new fraud in internet auctions ☆

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Abstract

We examine the effect of false-name bids on combinatorial auction protocols. False-name bids are bids submitted by a single bidder using multiple identifiers such as multiple e-mail addresses. The obtained results are summarized as follows: (1) the Vickrey–Clarke–Groves (VCG) mechanism, which is strategy-proof and Pareto efficient when there exists no false-name bid, is not false-name-proof; (2) there exists no false-name-proof combinatorial auction protocol that satisfies Pareto efficiency; (3) one sufficient condition where the VCG mechanism is false-name-proof is identified, i.e., the concavity of a surplus function over bidders.

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1. Introduction

Internet auctions have become an especially popular part of Electronic Commerce (EC). Various theoretical and practical studies on Internet auctions have already been conducted (Monderer and Tenenholtz, 2000a, 2000b; Sandholm, 1996; Wurman et al., 1998). Among these studies, those on combinatorial auctions have lately attracted considerable attention (Fujishima et al., 1999; Klemperer, 1999; Rothkopf et al., 1998; Sandholm, 1999).

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Although conventional auctions sell a single good at a time, combinatorial auctions sell multiple goods with interdependent values simultaneously and allow the bidders to bid on any combination of goods. In a combinatorial auction, a bidder can express complementary/substitutable preferences over multiple goods. By taking into account such preferences, economic efficiency can be enhanced.

Although the Internet provides an excellent infrastructure for executing combinatorial auctions, we must consider the possibility of new types of cheating. For example, a bidder may try to profit from submitting false bids under fictitious names such as multiple e-mail addresses. Such an action is very difficult to detect since identifying each participant on the Internet is virtually impossible. We call a bid made under a fictitious name a *false-name bid*. Also, we call a protocol is *false-name-proof* if truth-telling without using false-name bids is a dominant strategy for each bidder.

The problems resulting from collusion have been discussed by many researchers (McAfee and McMillan, 1987, 1992; Milgrom and Weber, 1982; Milgrom, 2000). Compared with collusion, a false-name bid is easier to execute on the Internet since getting another identifier such as an e-mail address is cheap. We can consider false-name bids as a very restricted subclass of collusion.

A concept called *group-strategy-proof* is proposed to study another restricted subclass of general collusion (Muller and Satterthwaite, 1985; Moulin and Shenker, 1996). As discussed in Section 5, group-strategy-proof and false-name-proof are independent concepts, i.e., a group-strategy-proof protocol is not necessarily false-name-proof, and vice versa.

In this paper, we analyze the effects of false-name bids on combinatorial auction protocols. The obtained results can be summarized as follows:

- The Vickrey–Clarke–Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), which is strategy-proof and Pareto efficient if there exists no false-name bid, is not false-name-proof.
- There exists no false-name-proof combinatorial auction protocol that satisfies Pareto efficiency.
- We identify one sufficient condition where the VCG mechanism is false-name-proof, i.e., a surplus function is *concave* over bidders.

In the rest of this paper, we first develop the model of a combinatorial auction in which false-name bids are possible (Section 2). Next, we examine the effect of false-name bids in combinatorial auctions (Section 3). Then, we show a sufficient condition where the VCG mechanism is false-name-proof (Section 4). Finally, we discuss the difference between false-name-proof protocols and group-strategy-proof protocols (Section 5).

2. Formalization

In this section, we formalize a combinatorial auction protocol in which false-name bids are possible. Our model is based on that presented in (Monderer and Tenenholtz, 2000a), but our model is modified to handle false-name bids.

Assume there is a set of bidders $N = \{1, 2, \ldots, n\}$. We assume agent 0 is an auctioneer, who is willing to sell a set of goods $A = \{a_1, a_2, \ldots, a_l\}$. Each bidder i has his/her preferences over the subsets of A. Formally, we model this by supposing that bidder i privately observes a type θ_i, which is drawn from a set Θ. We assume a *quasi-linear, private value* model with *no allocative externality*, defined as follows.

Definition 1. The utility of bidder i, when i obtains a subset of goods $B \subseteq A$ and a monetary transfer t_i, is represented as $v(B, \theta_i) + t_i$.

We assume evaluation value v is normalized by $v(\emptyset, \theta_i) = 0$. Also, we assume *free disposal*, i.e., $v(B', \theta_i) \geqslant v(B, \theta_i)$ for all $B \subseteq B'$. Furthermore, for auctioneer 0, for any subset of goods B, we assume $v(B, \theta_0) = 0$ holds.

To formalize false-name bids, we introduce a set of identifiers that bidders can use.

Definition 2. There exists a set of identifiers $M = \{\mathrm{id}_1, \mathrm{id}_2, \ldots, \mathrm{id}_m\}$. Furthermore, there exists a mapping function ϕ, where $\phi : N \to 2^M \setminus \{\emptyset\}$. 2^M is a power set of M. $\phi(i)$ represents a set of identifiers a bidder i can use. We assume for all i, $|\phi(i)| \geqslant 1$ and $\bigcup_i \phi(i) = M$ hold. Also, we assume for all $i \neq j$, $\phi(i) \cap \phi(j) = \emptyset$ holds, i.e., the identifiers of different bidders are mutually exclusive.

We assume $\phi(i)$ is also the private information of bidder i. Therefore, the set of signals are represented as $T = \Theta \times (2^M \setminus \emptyset)$, where the signal of bidder i is $(\theta_i, \phi(i)) \in T$.

In other words, a bidder can submit multiple bids pretending to be multiple bidders, but a bidder cannot impersonate another real, existing bidder. Also, the auctioneer 0 has no knowledge of ϕ and each bidder i only knows $\phi(i)$ and does not know $\phi(j)$ for $j \neq i$.

Next, we define a combinatorial auction protocol. For simplicity, we restrict our attention to *almost anonymous mechanisms*, in which obtained results are invariant under permutation of identifiers except for the cases of ties. We describe the condition that an almost anonymous mechanism must satisfy for the cases of ties later. Also, we restrict our attention to auction protocols, in which the set of messages for each identifier is $\Theta \cup \{0\}$, where 0 is a special symbol used for "non-participation." The fact that this restriction does not harm the generality of our results does not follow directly form the revelation principle, because the signal of a bidder is a pair $(\theta_i, \phi(i))$, and not just θ_i. We comment on this usage of the revelation principle at the end of this section.

Definition 3. A combinatorial auction protocol is defined by $\Gamma = (k(\cdot), t(\cdot))$. We call $k(\cdot)$ allocation function and $t(\cdot)$ transfer function. Let us represent a profile of types, each of which is declared under each identifier as $\theta = (\theta_{\mathrm{id}_1}, \theta_{\mathrm{id}_2}, \ldots, \theta_{\mathrm{id}_m})$, where $\theta_{\mathrm{id}_i} \in \Theta \cup \{0\}$. 0 is a special type declaration used when a bidder is not willing to participate in the auction:

$$k(\theta) = \big(k_0(\theta), k_{\mathrm{id}_1}(\theta), \ldots, k_{\mathrm{id}_m}(\theta)\big), \quad \text{where } k_{\mathrm{id}_i}(\theta) \subseteq A,$$
$$t(\theta) = \big(t_0(\theta), t_{\mathrm{id}_1}(\theta), \ldots, t_{\mathrm{id}_m}(\theta)\big), \quad \text{where } t_0(\theta), t_{\mathrm{id}_i}(\theta) \in \mathbb{R}.$$

$\mathbb{R}$ denotes the set of real numbers. Here, $t_0(\theta)$ represents the revenue of the auctioneer and $-t_{\mathrm{id}_i}(\theta)$ represents the payment of id_i.

We assume the following constraints are satisfied.

Allocation feasibility constraints: For all $i \neq j$, $k_{\mathrm{id}_i}(\theta) \cap k_{\mathrm{id}_j}(\theta) = \emptyset$, $k_{\mathrm{id}_i}(\theta) \cap k_0(\theta) = \emptyset$, and $k_{\mathrm{id}_j}(\theta) \cap k_0(\theta) = \emptyset$. Also, $\bigcup_{i=1}^{m} k_{\mathrm{id}_i}(\theta) \cup k_0(\theta) = A$.
Budget constraint: $t_0(\theta) = -\sum_{1 \leqslant i \leqslant m} t_{\mathrm{id}_i}(\theta)$.
Non-participation constraint: For all θ, if $\theta_{\mathrm{id}_i} = 0$, then $k_{\mathrm{id}_i}(\theta) = \emptyset$ and $t_{\mathrm{id}_i}(\theta) = 0$.

Also, in an almost anonymous mechanism, we assume for the cases of ties, the following condition is satisfied:

For a declared type profile $\theta = (\theta_{\mathrm{id}_1}, \theta_{\mathrm{id}_2}, \ldots, \theta_{\mathrm{id}_m})$, *if* $\theta_{\mathrm{id}_i} = \theta_{\mathrm{id}_j}$, *then* $v(k_{\mathrm{id}_i}(\theta), \theta_{\mathrm{id}_i}) + t_{\mathrm{id}_i}(\theta) = v(k_{\mathrm{id}_j}(\theta), \theta_{\mathrm{id}_j}) + t_{\mathrm{id}_j}(\theta)$ *holds.*

We define the fact that an allocation function is Pareto (or ex post) efficient as follows.

Definition 4. An allocation function $k(\cdot)$ is Pareto efficient if for all $k = (k_0, k_{\mathrm{id}_1}, \ldots, k_{\mathrm{id}_m})$, which satisfies the allocation feasibility constraints,

$$\sum_{1 \leqslant i \leqslant m} v\big(k_{\mathrm{id}_i}(\theta), \theta_{\mathrm{id}_i}\big) \geqslant \sum_{1 \leqslant i \leqslant m} v(k_{\mathrm{id}_i}, \theta_{\mathrm{id}_i})$$

holds. Let us denote a Pareto efficient allocation function as $k^*(\cdot)$.

A strategy of a bidder is defined as follows.

Definition 5. A strategy s of bidder i is a function $s : T \to (\Theta \cup \{0\})^M$ such that $s(\theta_i, \phi(i)) \in (\Theta \cup \{0\})^{|\phi(i)|}$ for every $(\theta_i, \phi(i)) \in T$. That is, $s(\theta_i, \phi(i)) = (\theta_{i,1}, \ldots, \theta_{i,m_i})$, where $\theta_{i,j} \in \Theta \cup \{0\}$ and $|\phi(i)| = m_i$.

We denote a profile of types for identifiers $M \setminus \phi(i)$ as $\theta_{\sim i}$. Also, we denote a profile of types for $\phi(i)$ declared by bidder i as $(\theta_{i,1}, \ldots, \theta_{i,m_i})$. Also, we denote a combination of these two type profiles as $((\theta_{i,1}, \ldots, \theta_{i,m_i}), \theta_{\sim i})$.

When a declared type profile is $\theta = ((\theta_{i,1}, \ldots, \theta_{i,m_i}), \theta_{\sim i})$, the utility of bidder i is represented as

$$v\big(sk_i(\theta), \theta_i\big) + st_i(\theta), \quad \text{where } sk_i(\theta) = \bigcup_{\mathrm{id}_j \in \phi(i)} k_{\mathrm{id}_j}(\theta),\ st_i(\theta) = \sum_{\mathrm{id}_j \in \phi(i)} t_{\mathrm{id}_j}(\theta).$$

We define a (weakly) dominant strategy of bidder i as follows.

Definition 6. For bidder i, a strategy $s^*(\theta_i, \phi(i)) = (s^*_{i,1}, \ldots, s^*_{i,m_i})$ is a dominant strategy if for all type profiles $\theta_{\sim i}$, $(\theta_{i,1}, \ldots, \theta_{i,m_i})$, where $\theta = ((s^*_{i,1}, \ldots, s^*_{i,m_i}), \theta_{\sim i})$, $\theta' = ((\theta_{i,1}, \ldots, \theta_{i,m_i}), \theta_{\sim i})$, $v(sk_i(\theta), \theta_i) + st_i(\theta) \geqslant v(sk_i(\theta'), \theta_i) + st_i(\theta')$ holds.

In a traditional setting where there exists no false-name bid, we say a direct revelation mechanism is truthfully implementable (in dominant strategies) or strategy-proof, when truthfully declaring his/her type is a dominant strategy for each bidder (Mas-Colell et al., 1995). On the other hand, in the problem setting in this paper, each bidder can submit multiple types in a mechanism. We define a mechanism/protocol is false-name-proof (or truthfully implementable in dominant strategies with the possibility of false-name bids) as follows.

Definition 7. We say a mechanism is false-name-proof when for all bidder i, $s^*(\theta_i, \phi(i)) = (\theta_i, 0, \ldots, 0)$ is a dominant strategy.

Since we assume a protocol/mechanism is almost anonymous, we can assume each bidder uses only the first identifier without loss of generality.

Bidder i can declare $(0, \ldots, 0)$, i.e., not participating in the auction. In this case, the utility of i becomes 0. Therefore, our definition that a mechanism is false-name-proof includes *individual rationality* (or *participation constraint*), which requires that the utility of each bidder must be non-negative.

In a traditional setting where there exists no false-name bid, the revelation principle guarantees that we can restrict our attention only to strategy-proof mechanisms without loss of generality. In the rest of this section, we discuss the meaning of the revelation principle when false-name declarations are possible.

When false-name bids are possible, the private information of each bidder is not only his/her type that determines the evaluation values of goods, but also a set of his/her identifiers $\phi(i)$. Therefore, in general, a direct revelation mechanism needs to ask not only a type, but also a set of identifiers a participant can use.

Formally, in a general mechanism, a social choice x is chosen from a set of alternatives X. We can assume a social choice function takes a set of pairs, where each pair consists of a type of each participant and a set of identifiers he/she can use and return a selected social choice, i.e., $f(\{(\theta_1, \phi(1)), \ldots, (\theta_n, \phi(n))\}) = x$.

It is rather straightforward to show that the revelation principle holds for such a social choice function. The revelation principle holds in a general mechanism, which is not necessarily almost anonymous.

Also, if we assume a mechanism is almost anonymous, there is no difference among identifiers, thus only the number of identifiers $m_i = |\phi(i)|$ affects the social choice. Furthermore, if we assume the social choice function f is invariant for the number of identifiers a participant can use, we can omit the declarations of identifiers. In this case, the revelation principle holds for a mechanism introduced in this section, which asks only a type of each participant and does not ask a set of identifiers.

3. The effect of false-name bids in the Vickrey–Clarke–Groves mechanism

In this section, we first examine the effect of false-name bids in the Vickrey–Clarke–Groves (VCG) mechanism.

The Vickrey–Clarke–Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), which is also called the generalized Vickrey auction protocol (Varian, 1995), is defined as follows:

Definition 8. In the VCG mechanism, $k^*(\cdot)$ is used for determining the allocation, and the transfer function is determined as follows.

$$t_{\mathrm{id}_i}(\theta) = \left[\sum_{j \neq i} v\big(k^*(\theta), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j \neq i} v\big(k^*_{-\mathrm{id}_i}(\theta), \theta_{\mathrm{id}_j}\big)\right],$$

where $k^*_{-\mathrm{id}_i}(\theta)$ is an allocation k that maximizes $\sum_{j \neq i} v(k_{\mathrm{id}_j}, \theta_{\mathrm{id}_j})$.

In short, in the VCG mechanism, each bidder is required to pay the decreased amount of the surplus, i.e., the sum of the gross utilities, of other bidders caused by his/her participation.

We describe how the VCG mechanism works in the following example.

Example 1. Three bidders (bidder 1, bidder 2, and bidder 3) are bidding for two goods, a_1 and a_2. The evaluation values of a bidder are represented as a triplet (the evaluation value for good a_1 only, the evaluation value for good a_2 only, the evaluation value for both a_1 and a_2):

bidder 1: $(7, 0, 7)$;
bidder 2: $(0, 0, 12)$;
bidder 3: $(0, 7, 7)$.

We assume there are four identifiers. Bidder 1 can use two identifiers, while bidders 2 and 3 can use only one identifier. If each bidder declares his/her true type using a single identifier, bidder 1 and bidder 3 will get good a_1 and good a_2, respectively. The payment of bidder 1 is calculated as $12 - 7 = 5$, since the sum of the gross utilities of other bidders when bidder 1 participates is 7, while the sum of the gross utilities of other bidders when bidder 1 does not participate would be 12. Bidder 3's payment is also equal to 5.

If there are no false-name bids, the VCG mechanism is strategy-proof, i.e., for each bidder, truthfully revealing his/her type is a dominant strategy. Now, we show an example where the VCG mechanism is not false-name-proof.

Example 2. Assume the same setting as the previous example, but the evaluation values of bidder 1 are different and bidder 3 is not interested in the auction:

bidder 1: $(7, 7, 14)$;
bidder 2: $(0, 0, 12)$;
bidder 3: $(0, 0, 0)$.

In this case, if bidder 1 uses a single identifier, he/she can obtain both goods, and the payment is equal to 12. However, bidder 1 can create the situation basically identical to Example 1 by using another identifier and splitting his/her bid. In this case, the payment becomes $5 + 5 = 10$. Therefore, for bidder 1, using a false-name bid is profitable.

Furthermore, the following non-existence theorem holds.

Proposition 1. *In combinatorial auctions, there exists no false-name-proof auction protocol that satisfies Pareto efficiency.*

Proof. We are going to prove the proposition by presenting a generic counter example assuming there exists a false-name-proof, Pareto efficient protocol.

Let us assume that there are two goods, a_1 and a_2, and three bidders denoted by bidder 1, bidder 2, and bidder 3. The evaluation values of a bidder are represented as a triplet: (the evaluation value for good a_1 only, the evaluation value for good a_2 only, the evaluation value for both a_1 and a_2). We assume there are four identifiers. Bidder 1 can use two identifiers, while bidders 2 and 3 can use only one identifier:

bidder 1: $(b, 0, b)$;
bidder 2: $(0, 0, b + c)$;
bidder 3: $(0, b, b)$.

Let us assume $b > c$. According to Pareto efficiency, bidder 1 gets good a_1 and bidder 3 gets good a_2. Let p_b denote the payment of bidder 1.

If bidder 1 declares his/her evaluation value for good a_1 as $b' = c + \epsilon$, the allocation does not change. Let $p_{b'}$ denote bidder 1's payment in this situation. The inequality $p_{b'} \leqslant b'$ should hold, otherwise, if bidder 1's true evaluation value for good a_1 were b', truth-telling would not be a dominant strategy since bidder 1 is not willing to participate if $p_{b'} > b'$. Furthermore, since truth-telling is a dominant strategy, $p_b \leqslant p_{b'}$ should hold. These assumptions lead to $p_b \leqslant c + \epsilon$. The condition for bidder 3's payment is identical to that for bidder 1's payment.

Next, we assume another situation where bidder 3 is not interested in the auction:

bidder 1: $(b, b, 2b)$;
bidder 2: $(0, 0, b + c)$;
bidder 3: $(0, 0, 0)$.

According to Pareto efficiency, both goods go to bidder 1. Let us denote the payment of bidder 1, p_{2b}. If bidder 1 uses a false-name bid and splits his/her bid, the same result as in the previous case can be obtained. Since the protocol is false-name-proof, the following inequality must hold, otherwise, bidder 1 can profit by using another identifier and splitting his/her bid: $p_{2b} \leqslant 2 \times p_b \leqslant 2c + 2\epsilon$.

On the other hand, let us consider the following case:

bidder 1: $(d, d, 2d)$;
bidder 2: $(0, 0, b + c)$;
bidder 3: $(0, 0, 0)$

Let us assume $c + \epsilon < d < b$, and $b + c > 2d$. According to Pareto efficiency, both goods go to bidder 2. Consequently, the utility of bidder 1 is 0. However, if bidder 1 declares his/her evaluation values as $(b, b, 2b)$ instead of $(d, d, 2d)$, both goods go to bidder 1 and the payment is $p_{2b} \leqslant 2c + 2\epsilon$, which is smaller than $2d$, i.e., bidder 1's true evaluation value of these two goods. Therefore, bidder 1 can increase the utility by overstating his/her true evaluation values.

Thus, in combinatorial auctions, there exists no false-name-proof auction protocol that satisfies Pareto efficiency. □

Please note that Proposition 1 holds in more general settings. The proof relies on the model defined in Definition 2.1, but it does not require the assumption of free disposal. Also, the proof does not rely on the fact that the mechanism is almost anonymous.

4. Sufficient condition where the VCG mechanism is false-name-proof

To derive a sufficient condition where the VCG mechanism is false-name-proof, we introduce the following function.

Definition 9. For a set of bidders and their types $Y = \{(y_1, \theta_{y_1}), (y_2, \theta_{y_2}), \ldots\}$ and a set of goods $B \subseteq A$, we define surplus function U as follows. Let us denote $K_{B,Y}$ as a set of feasible allocations of B to Y:

$$U(B, Y) = \max_{k \in K_{B,Y}} \sum_{(y_i, \theta_{y_i}) \in Y} v(k_{y_i}, \theta_{y_i}).$$

In particular, for a set of all goods A, we abbreviate $U(A, Y)$ as $U_A(Y)$.

Definition 10. We say $U_A(\cdot)$ is concave over bidders if for all possible sets of bidders Y, Z, and W, where $Y \subseteq Z$, the following condition holds:

$$U_A(Z \cup W) - U_A(Z) \leqslant U_A(Y \cup W) - U_A(Y).$$

Proposition 2. *The VCG mechanism is false-name-proof if the following conditions are satisfied*:

- Θ *satisfies that* $U_A(\cdot)$ *is concave for every subset of bidders with types in* Θ.
- *Each declared type is in* $\Theta \cup \{0\}$.

The proof of this proposition is relegated to Appendix A.

The second condition, i.e., the declared (not necessarily true) type also must be in $\Theta \cup \{0\}$, is required by the following reason. Even if bidders' true types satisfy the concavity condition, if bidder i declares a false type so that the concavity condition is violated (although doing so is not rational for bidder i), it is possible that using false-name bids is profitable for another bidder j.

First, we show one sufficient condition where the concavity of U_A is satisfied, i.e., the gross substitutes condition. The definition of this condition is as follows (Gul and Stacchetti, 1999; Kelso and Crawford, 1982).

Definition 11. Given a price vector $p = (p_{a_1}, \ldots, p_{a_l})$, we denote

$$D_i(p) = \left\{ B \subset A \colon v(B, \theta_i) - \sum_{a_j \in B} p_{a_j} \geqslant v(C, \theta_i) - \sum_{a_j \in C} p_{a_j},\ \forall C \subset A \right\}.$$

$D_i(p)$ represents the collection of bundles that maximize the net utility of bidder i under price vector p. Then, we say that the gross substitutes condition is satisfied, if for any two price vectors p and p' such that $p' \geq p$, $p'_{a_j} = p_{a_j}$, and $a_j \in B \in D_i(p)$, then there exists $B' \in D_i(p')$ such that $a_j \in B'$.

In short, the gross substitutes condition states that if good a_j is demanded with price vector p, it is still demanded if the price of a_j remains the same, while the prices of some other goods increase. The key property that makes the gross substitutes condition so convenient is that, in an auction that satisfies the gross substitutes condition, Walrasian equilibria exist (Kelso and Crawford, 1982).

One special case where the gross substitutes condition holds is a multi-unit auction, in which multiple units of an identical good are auctioned and the marginal utility of each unit is constant or diminishes.

Instead of showing directly the fact that the gross substitutes condition implies the concavity, we introduce another sufficient condition called *submodularity*.

We define U is submodular for a set of bidders as follows.

Definition 12. We say U is submodular for a set of bidders X, if the following condition is satisfied for all sets $B \subseteq A$ and $C \subseteq A$:

$$U(B, X) + U(C, X) \geqslant U(B \cup C, X) + U(B \cap C, X).$$

The following proposition holds.

Proposition 3. *If U is submodular for all set of bidders $X \subseteq N$, then U_A is concave.*

Proof. Let us choose three mutually exclusive subsets of bidders Y, Z', W. Also, let us assume in an allocation that maximizes $U(A, Y \cup Z' \cup W)$, B_Y, $B_{Z'}$, and B_W are allocated to Y, Z', and W, respectively. Since we assume free disposal, we can assume $A = B_Y \cup B_{Z'} \cup B_W$, i.e., each good is allocated to some bidder. The following formula holds:

$$U(A, Y \cup Z' \cup W) = U(B_Y, Y) + U(B_{Z'}, Z') + U(B_W, W).$$

Also, the following formula holds:

$$U(A, Y \cup W) \geqslant U(B_Y \cup B_{Z'}, Y) + U(B_W, W).$$

The right side represents the surplus when allocating $B_Y \cup B_{Z'}$ to bidders Y and B_W to bidders W. This inequality holds since the left side is the surplus of the best allocation including this particular allocation.

Similarly, the following formula holds:

$$U(A, Y \cup Z') \geqslant U(B_Y \cup B_W, Y) + U(B_{Z'}, Z').$$

By adding these two formulae, we obtain the following formula:

$$U(A, Y \cup W) + U(A, Y \cup Z') \geqslant U(B_Y \cup B_{Z'}, Y) + U(B_Y \cup B_W, Y) + U(B_{Z'}, Z') + U(B_W, W).$$

From the fact U is submodular, the following formula holds:

$$U(B_Y \cup B_{Z'}, Y) + U(B_Y \cup B_W, Y) \geqslant U(A, Y) + U(B_Y, Y).$$

From these formulae, we obtain the following formula:

$$U(A, Y \cup W) + U(A, Y \cup Z') \geqslant U(A, Y) + U(B_Y, Y) + U(B_{Z'}, Z') + U(B_W, W) \geqslant U(A, Y) + U(A, Y \cup Z' \cup W).$$

By setting $Z = Y \cup Z'$, we get $U_A(Z \cup W) - U_A(Z) \leqslant U_A(Y \cup W) - U_A(Y)$. □

The condition that U is submodular can be considered as a kind of a “necessary” condition for U_A to be concave, i.e., the following proposition holds.

Proposition 4. *If U is not submodular for a set of bidders X and a set of goods B and C, i.e., $U(B, X) + U(C, X) < U(B \cup C, X) + U(B \cap C, X)$, then we can create a situation where for a set of bidders Y, although U is submodular for Y, U_A is not concave for $X \cup Y$, where $A = B \cup C$.*

Proof. The proof of this proposition is relegated to Appendix B. □

Please note that the fact U is not submodular for some set of bidders X does not necessarily mean that the concavity condition will be violated for all situations that involve a set of bidders X. For example, if all bidders have all-or-nothing evaluation values for goods a_1 and a_2, i.e., for all i, $v(\{a_1\}, \theta_i) = v(\{a_2\}, \theta_i) = 0$, while $v(\{a_1, a_2\}, \theta_i) > 0$, clearly, U is not submodular, but we can create a situation where U_A is concave.

In (Gul and Stacchetti, 1999), it is shown that if evaluation value v for each bidder satisfies gross substitutes condition and monotonicity, then the surplus function U is submodular.[1] Therefore, if evaluation value v for each bidder satisfies the gross substitutes condition and monotonicity, which is satisfied if we assume free disposal, then U_A is concave.

Please note that as shown in (Gul and Stacchetti, 1999), even if evaluation value v for each bidder is submodular, it is not sufficient to guarantee that U is submodular.

[1] This can be derived from Theorem 6 and Lemma 1 in (Gul and Stacchetti, 1999).

5. Discussions

A concept called *group-strategy-proof* is proposed to study another restricted subclass of general collusion (Muller and Satterthwaite, 1985; Moulin and Shenker, 1996). An auction protocol is group-strategy proof if there exists no group of bidders that satisfies the following condition.

- By deviating from truth-telling, each member in the group obtains at least the same utility compared with the case of truth-telling, while at least one member of the group obtains a better utility compared with the case of truth-telling.

Group-strategy-proof and false-name-proof are independent concepts. Let us show an example where a protocol is false-name-proof, but not group-strategy-proof.

Let us assume there are two goods a_1 and a_2, and two bidders 1 and 2. We assume $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, where the evaluation values for these types are represented as follows:

- θ_1: $(10, 9, 18)$;
- θ_2: $(9, 10, 18)$;
- θ_3: $(10, 0, 10)$;
- θ_4: $(0, 10, 10)$.

Each of these types satisfies the gross substitutes condition, since when the number of goods are two, the fact that the evaluation value is subadditive, i.e., $v(\{a_1\}, \theta_i) + v(\{a_2\}, \theta_i) \geqslant v(\{a_1, a_2\}, \theta_i)$, implies the gross substitutes condition (Kelso and Crawford, 1982). Thus, as shown in the previous section, the VCG mechanism is false-name-proof in this example.

However, the VCG mechanism is not group-strategy-proof. Let us assume the type of bidder 1 is θ_1 and the type of bidder 2 is θ_2. By truth-telling, bidders 1 and 2 obtain a_1 and a_2, respectively, and each pays 8, thus the utility of each bidder is $10 - 8 = 2$. On the other hand, let us assume bidder 1 declares its type as θ_3 and bidder 2 declares its type as θ_4, i.e., both bidders understate their evaluation values of one good.

Then, the payment becomes 0, and the utility of each bidder becomes $10 - 0 = 10$. Thus, the utility of each bidder increases by deviating from truth-telling, i.e., the VCG mechanism is not group-strategy-proof in this case.

Next, let us show an example where a protocol is group-strategy-proof, but not false-name-proof. Let us assume an auction of a single-item, single-unit. Clearly, the following protocol is group-strategy-proof.

- The auctioneer sets a reservation price p. The winner is chosen randomly from the bidders whose declared evaluation value is larger than p. The winner pays p.

However, this protocol is not false-name-proof. A bidder can increase his/her chance of winning by submitting multiple bids. Please note that this protocol does not fit the definitions used in this paper, since this is a randomized mechanism.

6. Conclusions

We studied the effect of false-name bids on combinatorial auction protocols. We showed a formal model of combinatorial auction protocols in which false-name bids are possible. Then, we showed that the VCG mechanism is not false-name-proof. Furthermore, we showed a generalized counter-example that illustrates there exists no false-name-proof combinatorial auction protocol that satisfies Pareto efficiency. Furthermore, we identified one sufficient condition where the VCG mechanism is false-name-proof, i.e., the concavity of the surplus function over bidders.

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Appendix A. Concavity implies false-name-proof

To prove Proposition 2, we first prove the following proposition.

Proposition 5. *Assume $U_A(\cdot)$ is concave and the declared types also satisfy the concavity condition. Then, if a bidder uses two identifiers, the bidder can obtain more* (*or the same*) *utility by using a single identifier.*

Proof. Assume bidder 1 can use two identities id_1 and id_2. Also, let us assume bidder 1 declares type θ_{id_1} for id_1 and type θ_{id_2} for id_2.

Let us represent a type profile $\theta = (\theta_{\mathrm{id}_1}, \theta_{\mathrm{id}_2}, \theta_{\mathrm{id}_3}, \ldots, \theta_{\mathrm{id}_m})$, where $\theta_{\mathrm{id}_1}, \theta_{\mathrm{id}_2}, \theta_{\mathrm{id}_3}, \ldots, \theta_{\mathrm{id}_m}$ are declared types. The monetary transfer bidder 1 gets is the sum of:

$$t_{\mathrm{id}_1}(\theta) = \left[\sum_{j\neq 1} v\big(k^*(\theta), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j\neq 1} v\big(k^*_{-\mathrm{id}_1}(\theta), \theta_{\mathrm{id}_j}\big)\right]$$

and

$$t_{\mathrm{id}_2}(\theta) = \left[\sum_{j\neq 2} v\big(k^*(\theta), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j\neq 2} v\big(k^*_{-\mathrm{id}_2}(\theta), \theta_{\mathrm{id}_j}\big)\right].$$

Now, let us assume that bidder 1 uses only a single identifier id_1, and declares his/her type as θ'_{id_1}, so that the following condition is satisfied for all bundle B:

$$v\big(B, \theta'_{\mathrm{id}_1}\big) = v(B, \theta_{\mathrm{id}_1}) + v(B, \theta_{\mathrm{id}_2}).$$

Now, the declared type profile is $\theta' = (\theta'_{\mathrm{id}_1}, 0, \theta_{\mathrm{id}_3}, \ldots, \theta_{\mathrm{id}_m})$. Obviously, $k^*_{\mathrm{id}_1}(\theta) \cup k^*_{\mathrm{id}_2}(\theta) = k^*_{\mathrm{id}_1}(\theta')$ holds, i.e., the goods bidder 1 obtains do not change. The monetary transfer bidder 1 gets is as follows:

$$t_{\mathrm{id}_1}(\theta') = \left[\sum_{j\neq 1} v\big(k^*(\theta'), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j\neq 1} v\big(k^*_{-\mathrm{id}_1}(\theta'), \theta_{\mathrm{id}_j}\big)\right].$$

We are going to prove that $t_{\mathrm{id}_1}(\theta') \geqslant t_{\mathrm{id}_1}(\theta) + t_{\mathrm{id}_2}(\theta)$, i.e., the monetary transfer becomes larger when bidder 1 uses one identifier.

Let us denote $Y = \{(\mathrm{id}_3, \theta_{\mathrm{id}_3}), \ldots, (\mathrm{id}_m, \theta_{\mathrm{id}_m})\}$:

$$\begin{aligned}
&t_{\mathrm{id}_1}(\theta') - t_{\mathrm{id}_1}(\theta) - t_{\mathrm{id}_2}(\theta)\\
&\quad= \left[\sum_{j\neq 1} v\big(k^*(\theta'), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j\neq 1} v\big(k^*_{-\mathrm{id}_1}(\theta'), \theta_{\mathrm{id}_j}\big)\right]\\
&\qquad - \left(\left[\sum_{j\neq 1} v\big(k^*(\theta), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j\neq 1} v\big(k^*_{-\mathrm{id}_1}(\theta), \theta_{\mathrm{id}_j}\big)\right]\right)\\
&\qquad - \left(\left[\sum_{j\neq 2} v\big(k^*(\theta), \theta_{\mathrm{id}_j}\big)\right] - \left[\sum_{j\neq 2} v\big(k^*_{-\mathrm{id}_2}(\theta), \theta_{\mathrm{id}_j}\big)\right]\right)\\
&\quad= U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1}), (\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - v\big(k^*(\theta), \theta_{\mathrm{id}_1}\big) - v\big(k^*(\theta), \theta_{\mathrm{id}_2}\big) - U_A(Y)\\
&\qquad - \big(U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1}), (\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - v\big(k^*(\theta), \theta_{\mathrm{id}_1}\big)\big) + U_A\big(Y \cup \big\{(\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big)\\
&\qquad - \big(U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1}), (\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - v\big(k^*(\theta), \theta_{\mathrm{id}_2}\big)\big) + U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1})\big\}\big)\\
&\quad= U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1})\big\}\big) + U_A\big(Y \cup \big\{(\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - U_A(Y)\\
&\qquad - U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1}), (\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big).
\end{aligned}$$

By the concavity condition, the following formula is satisfied:

$$\begin{aligned}
&U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1}), (\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1})\big\}\big)\\
&\quad \leqslant U_A\big(Y \cup \big\{(\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - U_A(Y).
\end{aligned}$$

By transposition, we get,

$$\begin{aligned}
0 &\leqslant U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1})\big\}\big) + U_A\big(Y \cup \big\{(\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big) - U_A(Y)\\
&\quad - U_A\big(Y \cup \big\{(\mathrm{id}_1, \theta_{\mathrm{id}_1}), (\mathrm{id}_2, \theta_{\mathrm{id}_2})\big\}\big).
\end{aligned}$$

Therefore, we obtain

$$t_{\mathrm{id}_1}(\theta') \geqslant t_{\mathrm{id}_1}(\theta) + t_{\mathrm{id}_2}(\theta). \qquad \square$$

By repeatedly applying Proposition 5, we can show that if a bidder is using more than two identities, he/she can obtain more (or the same) utility by using a single identifier.

Appendix B. Proof of Proposition 4

We assume for a set of bidders X and a set of goods B and C, where $B \cup C = A$, $U(B, X) + U(C, X) < U(B \cup C, X) + U(B \cap C, X)$ holds.

Let us assume for each good $a_i \in A \setminus B$, there exists bidder $i \notin X$, who is interested in only a_i, and his/her evaluation value for a_i is larger than $U(A, X)$. Let us denote a set of these bidders as W. Similarly, let us assume for each good $a_j \in A \setminus C$, there exists bidder $j \notin X$, who is interested in only a_j, and his/her evaluation value for a_j is larger than $U(A, X)$. Let us denote a set of these bidders as Z. It is clear that U is submodular for $W \cup Z$, since these bidders are *unit demand consumers*, and satisfies the gross substitutes condition (Gul and Stacchetti, 1999).

It is clear that in the allocation that maximizes $U_A(X \cup W \cup Z)$, $A \setminus B$ are allocated to bidders in W, $A \setminus C$ are allocated to bidders in Z, and $B \cap C$ are allocated to bidders in X.

Also, for $X \cup W$, it is clear that in the allocation that maximizes $U_A(X \cup W)$, $A \setminus B$ are allocated to bidders in W, and B are allocated to bidders in X. Similarly, in the allocation that maximizes $U_A(X \cup Z)$, $A \setminus C$ are allocated to bidders in Z, and C are allocated to bidders in X.

Thus, the following formulae hold:

$$U_A(X \cup W) = U(A \setminus B, W) + U(B, X),$$
$$U_A(X \cup Z) = U(A \setminus C, Z) + U(C, X),$$
$$U_A(X \cup W \cup Z) = U(A \setminus B, W) + U(A \setminus C, Z) + U(B \cap C, X).$$

From these formulae and the assumption $U(B, X) + U(C, X) < U(B \cup C, X) + U(B \cap C, X)$, the following formula holds:

$$\begin{aligned} &U_A(X \cup W) + U_A(X \cup Z) \\ &\quad = U(A \setminus B, W) + U(B, X) + U(A \setminus C, Z) + U(C, X) \\ &\quad = U_A(X \cup W \cup Z) - U(B \cap C, X) + U(B, X) + U(C, X) \\ &\quad < U_A(X \cup W \cup Z) + U(B \cup C, X) = U_A(X \cup W \cup Z) + U_A(X). \end{aligned}$$

Thus, concavity is violated since $U_A(X \cup W) - U_A(X) < U_A(X \cup Z \cup W) - U_A(X \cup Z)$.

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