The Exact Cost of Exploring Streets with a CAB

Extended Abstract*

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Abstract

A fundamental problem in robotics is to compute a path for a robot from its current location to a given target. In this paper we consider the problem of a robot equipped with an on-board vision system searching for a target t in an unknown environment.

We assume that the robot is located at a point s in a polygon that belongs to the well investigated class of polygons called *streets*. In this paper we present the first *exact* analysis of the *continuous angular bisector* (*CAB*) strategy, which has been considered several times before, and show that it has a competitive ratio of ≈ 1.6837 in the worst case.

1 Introduction

Finding a path from a starting location to a target within a given scene is an important problem in robotics. A natural and realistic setting is to assume that the robot has only partial knowledge of its surroundings and that the amount of information available to the robot increases as it travels and discovers its surroundings. For this purpose, the robot is equipped with an on-board vision system that provides the visibility map of its local environment. The robot uses this information to devise a search path for a visually identifiable target located outside the current visibility region. A search strategy is called *c-competitive* if the path traveled by the robot to find the target is at most c times longer than a shortest path. The parameter c is called the *competitive ratio* of the strategy.

As can easily be seen, there is no strategy with a competitive ratio of $o(n)$ for scenes with arbitrary obstacles having a total of n vertices [2] even if we restrict ourselves to searching in a simple polygon. Therefore, the on-line search problem has been studied previously for the case where the geometry of the terrain is restricted to searching in special classes of simple polygons [4, 5, 8, 16, 17].

In this paper we study the continuous angular bisector (CAB) strategy to search in *street polygons*. In a street P the starting point s and the target t are located on the boundary of P and all points in P are visible from some point on the shortest path from s to t.

2 Previous Work

The class of street polygons was first introduced by Klein, and he was also the first to present a search strategy for streets [9]. His strategy *lad* is based on the idea of minimizing the *l*ocal *a*bsolute *d*etour. He gives an upper bound on its competitive ratio of $1 + 3/2\pi\,(\sim 5.71)$, later improved by Icking to $1 + \pi/2 + \sqrt{1 + \pi^2/4}$ (~ 4.44) [6].

A number of other strategies have been presented since by Kleinberg [10], López-Ortiz and Schuierer [13, 14, 15], Semrau [18], Dasgupta *et al.* [3], and Kranakis and Spatharis [11]. Unfortunately, the analyses of the last two results turned out to be erroneous. The currently best known competitive ratio is ≈ 1.51 [7].

It is well known that there is no strategy with a competitive ratio less than $\sqrt{2}$ [9]. In this paper we present the first exact analysis of the strategy *continuous angular bisector* (*CAB*) which has been considered independently by several authors both in its continuous [3, 12, 15] and discrete form [9, 12]. We show that the competitive ratio of *CAB* is ≈ 1.6837. The previously best known upper bound on the competitive ratio is 2.03 [15]. López-Ortiz shows a lower bound of ≈ 1.6837 on the competitive ratio of *CAB* if only triangles are considered [12]. We show that *CAB* is no worse even in general streets.

CAB is a very natural strategy where the robot walks on a curve such that, at any moment, the direction it is facing always bisects its visibility angle. It is somewhat surprising that *CAB* can be analysed exactly as it consists of hyperbolic arcs whose length can not be expressed in a closed form. The importance of *CAB* is threefold:

- it compares favourably to most other strategies proposed [9, 10, 13, 14, 15],
- it is a C^1-*continuous* strategy, as opposed to all others which contain bends; thus, a robot may follow a *CAB* path without having to stop, and
- is used as a component of hybrid strategies for searching in streets as well as other domains, such as, for example, to search for the kernel of a polygon [8].

3 Searching with a CAB

We assume that the robot is modeled by a point in a simple polygon P in the plane. Its start position is s and the position

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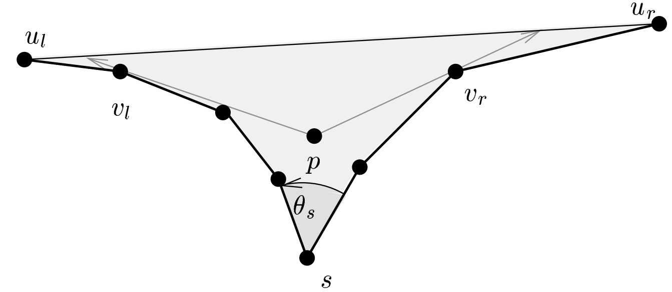

Figure 1: A funnel polygon.

of the target is denoted by t, both of which are vertices of P. The following facts are known (see for example [9, 13, 18]):

The competitive ratio of most strategies—and, in particular, *CAB*—depends only on the competitive ratio that they achieve in funnels. A *funnel* is a street which consists of two reflex chains and one line segment. The point common to both reflex chains is denoted by s(see Figure 1). The two reflex chains end in two vertices u_l and u_r whose connecting line segment l closes the polygon. The target t is hidden either at u_l or at u_r. Lastly, if a strategy achieves a competitive factor c in funnels, then it can easily be extended to a c-competitive strategy for searching in streets [9].

While a strategy proceeds, we always denote the most advanced visible point on the left chain with v_l and the most advanced visible point on the right chain with v_r. Let $\theta_p = \angle v_l p v_r$.

Suppose that the robot moves from the point p to a point q in the triangle formed by v_l, p, and v_r. Let α be the angle $\angle p v_l q$ and β be the angle $\angle q v_r p$, then the new visibility angle θ_q at q is given by $\theta_q = \theta_p + \alpha + \beta$.

> ***CAB* strategy:** The robot walks along the curve $\mathcal{C}$ such that, for all points p on $\mathcal{C}$, the direction of the motion of the robot (that is the tangent to $\mathcal{C}$ in p) bisects the angle $\angle v_l p v_r$. [3, 8, 12].

Note that one of the points v_l and v_r changes when the path of the robot intersects a line that is collinear with one of the edges of the funnel. However, the visibility angle does not change. Hence, the curve generated by *CAB* is a C^1-continuous curve composed of hyperbolic arcs with foci at v_l and v_r. Let $d(p, q)$ be the length of the shortest path from p to q in P and $|pq|$ the length of the line segment from p to q. The points p that lie on the path generated by *CAB* satisfy the equation $|pv_l| - |pv_r| = d(s, v_l) - d(s, v_r)$.

The analysis proceeds by considering a discretization of the bisector strategy. That is the robot determines the bisector for its current position p and moves a small amount along that bisector until it reaches a point p', where it again determines the bisector and moves on it. Eventually the robot locates the target and moves towards it.

Note that when the distance traversed in each bisector goes to zero, this strategy converges to *CAB*. With the aid of trigonometry, and using the very particular properties of *CAB* described above, one can establish equations for the detour of the robot on a single step. The sum of all these steps gives the distance extra traversed by the robot. These summations, when considered in the limit and in the worst case, reduce to Riemann summations that result in tractable integrals. Indeed the worst case detour is given by

$$\max_{0 \le \phi \le \pi} \left(\sin^2 \frac{\phi}{2} \int_{\phi}^{\pi} \frac{\tan(\theta/4)}{\sin^2(\theta/2)}\, d\theta \right) =$$

$$= \max_{0 \le \phi \le \pi} \left(\sin^2 \frac{\phi}{2} \left(1 - \ln \tan \frac{\phi}{4} \right) - 1 + \cos \frac{\phi}{2} \right) \approx 0.68372.$$

The competitive ratio is the normalized total distance traversed by the robot, including the detour, that is $1 + 0.68372 = 1.68372$, as claimed. This value is exact as there is a matching lower bound by López-Ortiz in [12].

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