Adaptive Searching in One and Two Dimensions

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Abstract

1 Introduction

Searching in a geometric space is an active area of research, predating computer technology. The applications are varied ranging from robotics, to search-and-rescue operations in the high seas [RS71, LOS01b] as well as in land, such as in an avalanche [BHH$^+$02] or an office space [HIKK01, DHS95a, DHS95b, Ick94], to scheduling of heuristic algorithms for solvers searching an abstract solution space for a specific solution [KMSY94, KRT93, LOS02, ALOH08, LOAH06]. Within academia, the field has seen two marked boosts in activity. The first was motivated by the loss of weaponry off the coast of Spain in 1966 in what is known as the Palomares incident and of the USS Thresher and Scorpion submarines in 1963 and 1966 respectively [RS71, Sto89]. A second renewed thrust took place in the late 1980s when the applications for autonomous robots became apparent.

Geometric searching has proved a fertile ground within computational geometry for the design and analysis of search and recognition strategies under various initial conditions [IKM93, HIKK01, CL93, DHS95a, DI94b, LOS96c, LOS96d, LO96, LOS95b, LOS95a, LOS96b].

The basic search scenarios consist of exploring a one dimensional object, such as a path or office corridor, usually modeled as the real line, and of exploring a two dimensional scene, such as a room or a factory floor, usually modelled as a polygonal scene. However, in spite of numerous advances in the theoretical understanding of both of these scenarios, so far such solutions have generally had a limited impact in practice.

Over the years various efforts have been made to address this situation, both in terms of isolated research papers attempting to narrow the gap, as well as in organized efforts such as the Algorithmic Foundations of Robotics conference and the Dagstuhl seminars on on-line robotics which bring together theoreticians and practitioners. From these it is apparent that the cost model and hence the solutions obtained from theoretical analysis do not fully reflect real life constraints. Several efforts have been made to resolve this, such as including the turn cost, the scanning cost, and error in navigation and recknoning [DFG06, FKN06, Kam05, LOS95b].

In this paper we address one more shortcoming of the standard model, which is the emphasis on the worst case scenario. Consider for example a vacuuming robot—such as Roomba(TM). Such a robot explores the environment using sophisticated motion planning algorithms with the goal of attaining complete coverage of the floor surface within a reasonable amount of time. It is not hard to device worst case floor plans (such as complex mazes) which would not be covered very efficiently. In practice this is not a concern since (i) most rooms in practice are relatively simple and (ii) if the robot ever encounters such a complex scene a drop in performance is only to be expected and users would not mind a severe degradation in performance. This naturally leads to the concept of adaptive algorithms, in which the performance is expressed in terms of the input size as well as a measure of difficulty of the input. That is, on simpler inputs the robot must perform more efficiently than on more complex ones.

In this paper we consider adaptive analysis of two basic geometric primitives: searching on the real line and looking around the corner.

Searching on the real line consists of finding a setting in which a point robot is imagined to stand at the origin of the real line wich contains a target t at an unknown distance to the d. The robot can only detect t if it stands on top of it. It can be shown that an optimal strategy under the competitive ration and in the worst case visits the rays under a doubling strategy with competitive ratio of 9 [Bec64, Gal80, BYCR93, LOS01a]. We refer the reader to the survey of Alpersen and Gal for a thorough discussion. Searching has proven to be a very useful tool for searching in a number of classes of simple polygons, such as star-shaped polygons [LOS97], generalized streets [DI94a, LOS96a], HV-streets [DHS95b], and θ-streets [DHS95b, Hip94].

However upon being presented by the optimal doubling strategy practitioners routinely report that they find the answer non-intuitive and generally “not optimal”. This holds both for the optimal strategy for either the average or the worst case. If we consider exploration as a valuable task, then the goal is no longer to solely minimize the time to the target, but to maximize the amount of information gained during the search. In the first case study we consider this case and obtain a strategy that is, sujectively, more pleasing to practitioners.

Geometric planning algorithms are usually analyzed

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using competitive analysis [ST85] framework. In this approach, an online algorithm (an algorithm with incomplete information) is compared with an optimal offline algorithm (an algorithm with complete information). An algorithm has competitive ratio c if its performance is at most c times worse than the optmal offline algorithm (plus an additive constant). Icking et al. [IKM93] provided an algorithm with competitive ratio $c \approx 1.21218$ and proved that this is the best competitive ratio possible.

Competitive analysis is a worst-case measure and sometimes leads to pessimistic results. For example in geometric search we compare an online algorithm (an autonomos robot) with incomplete information to an algorithm with complete information on all possible configurations of the problem. In other words the adversary is too powerful in this setting. Also in real life, we usually encounter configurations that are not.......(talk about usual robot navigation in practice, Mandelbrot,....)

We start refining the competitive analysis framework by applying some ideas from adaptive analysis to an elementary geometric serach problem, namely looking around the corner. In adaptive analysis, a competitive algorithm has good performance on "good" inputs (these might be almost sorted sequences for sorting or usual configurations for robot navigation) and their performance on "bad" sequences is not too bad. This is usually done by normalizing the performance by the difficulty of teh instance. In practical robot navigation most corners have angles close to $\pi/2$ and usually we do not have angles close to 0 or π. As a first attemp for applying adaptive analysis ideas we consider $1/\sqrt{\sin\phi}$ as difficulty measure for looking around teh corner instances.

2 Problem Definition

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